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A Transient Two-Dimensional Finite Volume Model for the Simulation of Vertical U-Tube Ground Heat Exchangers

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ABSTRACT

The ability to predict both the long-term and short-term behavior of ground loop heat exchangers is critical to the design and energy analysis of ground source heat pump systems. A numerical model for the simulation of transient heat transfer in vertical ground loop heat exchangers is presented. The model is based on a two-dimensional fully implicit finite volume formulation. Numerical grids have been generated for different pipe sizes, shank spacing and borehole sizes using an automated parametric grid generation algorithm. The numerical method and grid generation techniques have been validated against an analytical model. The model has been developed with two main purposes in mind. The first application is use in a parameter estimation technique used to find the borehole thermal properties from short time scale test data. The second application is the calculation of nondimensional temperature response factors for short time scales that can be used in annual energy simulation.

INTRODUCTION

Ground source heat pump systems have become increasingly popular for both residential and commercial heating and cooling applications. These systems have been recognized to provide viable, environment-friendly alternatives to conventional unitary systems. They can make significant contributions to reductions in electrical energy usage, and allow more effective demand-side management. However, compared to air source heat pump systems, ground source systems have not been so widely used. This may be attributed to comparatively higher installation cost and ground area requirements but may also be attributed to the lack of reliable system design and simulation models. Even though the system design process for ground loop heat exchangers has passed the stage of heuristic models, Cane and Forgas (1991) estimate that current North American practice results in ground loop heat exchanger lengths being oversized by about 10% to 30%—a high enough percentage to make the short-time economics of these applications comparatively unattractive. Nevertheless, tens of thousands of residential systems per year are installed worldwide, and a steadily increasing interest in systems for nonresidential applications is observed. The further acceptance of the technology will also depend on the availability of accurate, reliable, and fast system design and simulation tools.

In ground source heat pump systems, heat is extracted from or rejected to the ground via either an open loop through which ground water is drawn off, or a closed loop through which water or a water/antifreeze solution circulates. Open loop systems are not considered here. The ground loop heat exchangers used in closed loop systems can be either horizontal or vertical in configuration. The model presented is applicable to vertical borehole ground loop heat exchangers. The horizontal cross-section of a typical borehole is depicted in Figure 1.

Vertical ground loop heat exchangers typically consist of high-density-polyethylene (HDPE) pipe U-tubes inserted into 100 ft (30 m) to 300 ft (90 m) deep boreholes. The boreholes have typical diameters of 3″ (76 mm) to 5″ (127 mm) and the pipe diameters in the range from $\frac{3}{4}$″ (19 mm) to 1 $\frac{1}{2}$″ (38 mm). The pipes forming the U-tube are accordingly closely spaced in the borehole. A grout mixture is typically pumped into the borehole to fill the gap between the U-tube and the borehole walls. The purpose of the grout is to

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improve the heat transfer between the soil and plastic pipes by providing a better contact surface between them, and also to provide a seal around the U-tube to guard against migration of contaminants into the ground water system.

GROUND LOOP HEAT EXCHANGER MODELS

The viability of a particular ground loop heat exchanger design often depends on its ability to reject heat over a number of years and the avoidance of excessive buildup or loss of heat in the borehole field. A model for the design of ground loop heat exchanger fields has therefore to be computationally efficient enough to allow calculation of transient effects over long time scales. The use of an analytical model is attractive in terms of computational efficiency but the fact that the pipes are not coaxial with the borehole, and a number of different materials are involved, makes the task of finding a suitable analytical model difficult or impossible. However, a number of design methods using analytical approaches have been developed, but which make a number of simplifying assumptions. The most significant of these is the assumption of the so-called ''equivalent diameter'' approximation that treats the two legs of the U-tube as a single pipe coaxial with the borehole so that a solution such as the ''cylinder source'' solution (Carslaw and Jaeger 1947) may be applied. The geometry can alternatively be further approximated as an infinitely long ''line source'' (Kelvin 1882; Ingersoll 1954). Cane and Forgas (1991) give a review of a number of these models.

When line source and/or cylindrical source models are implemented in actual loop design models further adjustments are necessary to account for the leg-to-leg thermal short-circuiting effects and pipe wall and contact resistances. In the later phases of the life of a borehole field, where the buildup of heat in the far field is of much more significance than the heat distribution local to the borehole, these simplifications are correspondingly insignificant. Correspondingly, in the shorter term (of the order of hours to weeks) effects local to the borehole, and the influence of the geometry, are important. Numerical models of conduction around the borehole accordingly have the advantage of being able to account for the complexities of the geometry but the disadvantage of being computationally more expensive and are therefore more suitable for modeling on shorter time scales. A number of ground loop heat exchanger design methods have been devised that combine numerical and analytical methods and some of these are discussed below.

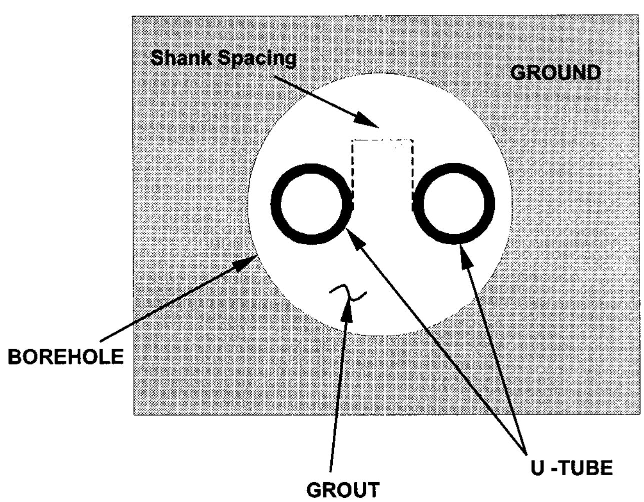

Figure 1 *Cross-section of a typical borehole.*

Eskilson's (1987) approach to the problem of determining the temperature response of a multiple borehole ground loop heat exchanger is based on dimensionless temperature response factors, called g functions. (These should not be confused with the g functions used in the cylinder source solution.) The response factors are computed with a two step process. First, a two-dimensional (radial-axial) explicit finite-difference simulation of a single borehole is performed to determine the response to a unit step function heat pulse. The borehole in the finite difference model has a finite length and diameter. The borehole (pipe and grout) thermal resistance and capacitance are neglected in the numerical model; the borehole thermal resistance is accounted for separately. Using the spatial temperature distribution of a single borehole, a spatial superposition is formed to determine the response of a predefined configuration of boreholes (characterized by their ratio of horizontal spacing to depth) to the unit step function heat pulse. When the borehole outer wall temperature vs. time response is nondimensionalized, the resulting dimensionless temperature vs. dimensionless time curve is the g function. Individual response factors, which give the temperature response to a specific length unit step function heat pulse, are determined by interpolating the g function.

Once the response factors have been determined, the response of the ground loop heat exchanger to any heat rejection/extraction vs. time profile can be determined by decomposing the heat rejection/extraction vs. time profile into a set of unit step functions. Then, the response of the ground loop heat exchanger to each unit step function can be superimposed to determine the overall response. Additional details, e.g., converting from nondimensional units to dimensional units, and accounting for the borehole thermal resistance are described by Eskilson (1987). The model is intended to provide the response of the ground to heat rejection/extraction over longer periods of time (up to 25 years) but as the numerical model that provides the g functions does not account for the local borehole geometry, it cannot accurately provide the shorter term response.

Hellstrom (1989, 1991) developed a simulation model for vertical ground heat stores, which are densely packed ground loop heat exchangers used for seasonal thermal energy storage. This type of system may or may not incorporate heat pumps. The model subdivides the ground volume with multiple boreholes into two regions. The volume that immediately surrounds a single borehole is described as the ''local'' region. The difference between the ''local'' average temperature and the average fluid temperature in the borehole for a given time is proportional to the heat rejection/extraction rate for that time via a time-dependent fluid-to-ground resistance. This is used to account for heat transfer conditions around individual boreholes due to short-time thermal variations. Over longer time scales, the heat flow field in this region does not change with time. A constant temperature difference is then computed due to the constant heat flux via a constant steady-flux thermal resistance. The second region is concerned with the heat conduction problem between the bulk of the heat store volume (multiple boreholes) and the far field. Hellstrom defines this to be the ‘‘global’’ problem. The ‘‘global’’ problem is treated as three components: a steady-state heat loss component, a thermal buildup component, and a periodic heat loss component. The steady-state regime for the ‘‘global’’ problem may be reached after several years (depending on the size of the heat store and heat rejection and extraction rates) during which a transient thermal buildup is assumed to occur around the borehole field where the heat flow gradually approaches a steady-state value.

Hellstrom’s model thus represents the total change in the initial ground temperature for a time step first by the spatial superposition of three parts: a so-called ‘‘global’’ temperature difference, a temperature difference from the ‘‘local’’ solution immediately around the heat store volume, and a temperature difference from the ‘‘local’’ steady-flux part. The ground temperature at any subsequent time is determined by decomposing the time-varying heat transfer profile into a series of individual step heat pulses and superimposing the resulting responses in time. The model is essentially a hybrid model that uses a numerical solution for the ‘‘local’’ and the ‘‘global’’ problems and then superimposes them spatially with the analytical solution from the steady-flux part. The numerical model uses a two-dimensional explicit finite difference scheme on the radial-axial coordinate system for the ‘‘global’’ problem. For the local solution, a one-dimensional radial mesh is used that divides the storage region into several sub-areas. Hellstrom’s model is not ideal for determining long time-step system responses for ground source heat pump systems since the geometry of the borehole field is assumed to be densely packed, with a minimum surface area to volume ratio, as is typical for heat stores.

Mei and Emerson (1985) developed a numerical model suitable for horizontal coils that included a model of the effects of frozen soil around the pipe. Their numerical model used an explicit finite difference scheme to solve three one-dimensional partial differential equations describing conduction radially through the pipe, frozen soil region, and far field region. These equations were coupled to a fourth one-dimensional partial differential equation representing the flow of heat along the pipe, resulting in a quasi–two-dimensional model. The model used different time steps for the pipe wall and the frozen soil region, and another significantly larger time step for the fluid and unfrozen soil region. The size of the frozen region at each position along the pipe was extended or contracted accordingly throughout the simulation. Mei and Emerson reported comparisons with experimental data over a 48-day simulation period.

Muraya (1995) and Muraya et al. (1996) used a transient two-dimensional finite element model to investigate the thermal interference between the U-tube legs. The model attempts to quantify this interference by defining a heat exchanger effectiveness based on soil and grout properties, shank spacing, far-field and loop temperatures, and heat dissipation rates. The model validation is conducted against two different applications of the analytical cylinder source solution using constant temperature and constant flux approaches. Based on the parametric studies performed, they were able to define an overall thermal effectiveness and a backfill effectiveness both of which depended on the borehole geometry.

Rottmayer *et al.* (1997) developed a numerical U-tube heat exchanger model based on an explicit finite-difference technique. A two-dimensional finite difference formulation on a polar grid was used to calculate the lateral heat transfer over each 10 ft (3 m) vertical section of the borehole. Conduction in the vertical direction was neglected but each section of the model was coupled via the boundary conditions to a model of flow along the U-tube. In this way a quasi–three-dimensional model was produced that could account for the variations in fluid temperature with depth. The geometry of the circular U-tube pipes were approximated by a ‘‘pie-sector’’ shape by matching the perimeter of the modeled noncircular tube to the actual circular pipe perimeter. They found that the model under-predicted the heat transfer from the U-tube by approximately 5% when compared to an analytical model and attributed this to the simplified pipe geometry. To account for this they introduced a ‘‘geometry factor’’ of the order of 0.3—0.5 which was used to modify the soil and grout components of the fluid to grout resistances in the finite difference equations. This size of this factor was determined so that the model gave the same steady state heat transfer rates as the analytical model.

The numerical model reported here has been developed with two purposes in mind. The first is to find the borehole thermal properties from short time scale test data by an inverse method (Austin 1998; Austin et al. 2000). In this technique the numerical model is used in a parameter estimation algorithm where the soil and grout thermal conductivities are the parameters to be estimated. The objective function is defined as the sum of the mean square errors between the test data temperatures and those predicted by the numerical model for a given set of parameter values. By minimizing the objective function the ‘‘best’’ values of the soil and grout thermal conductivities are found.

The long-term response of borehole fields is commonly described by nondimensional response curves known as ‘‘*g* functions’’ (Eskilson 1987). These are used to describe the performance of a particular borehole field configuration over the time scale of a single month to several years. The second purpose for the numerical model is to calculate the response to pulses of heat and derive similar nondimensional response curves for periods up to one month (Yavuzturk and Spitler 1999). Such response curves can provide a computationally sufficient method of determining the borehole thermal response for system energy simulation.

THE NUMERICAL METHOD

Transient heat conduction in the ground loop heat exchanger is represented here in two dimensions. A two-dimensional (horizontal) representation is reasonable if a number of assumptions are made:

- Three-dimensional effects at the ground surface and end of the U-tube are neglected.
- Inhomogeneities in the ground properties are neglected.
- The effects of changing pipe temperature with depth are approximated.

The transient conduction equation in polar coordinates can be expressed as

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}. \tag{1}$$

This equation has been discretized using a fully implicit finite volume approach (Patankar 1980, 1991). The typical arrangement of the finite volume cells on the polar grid is shown in Figure 2.

First order backwards differencing in time and second order central differencing in space have been used. The resulting discrete equation can be expressed as

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b \tag{2}$$

where

$$a_P = a_E + a_W + a_N + a_S + a_P^0 \tag{3}$$

and the coefficients a_n and b are defined as follows:

$$a_P^0 = \frac{(\rho c)_P \Delta V}{\Delta t}, \tag{4}$$

$$a_E = \frac{k_e \Delta r_e}{r_e(\delta\theta)_e}, \tag{5}$$

$$a_W = \frac{k_w \Delta r_w}{r_w(\delta\theta)_w}, \tag{6}$$

$$a_N = \frac{k_n r_n \Delta\theta_n}{(\delta r)_n}, \tag{7}$$

$$a_S = \frac{k_s r_s \Delta\theta_s}{(\delta r)_s}, \tag{8}$$

$$b = S_C \Delta V + a_P^0 T_P^0. \tag{9}$$

The resulting algebraic equations are linear (the material properties being kept constant in this case) and are solved reasonably efficiently using a line-by-line tridiagonal matrix algorithm with block correction (Patankar 1991).

PARAMETRIC GRID GENERATION

An algebraic algorithm has been developed to automatically generate numerical grids in polar coordinates for the ground loop heat exchanger geometry. This allows grids to be automatically generated for a variety of borehole diameters, pipe diameters, and pipe shank spacing combinations by varying a few control parameters. Advantage is taken of the symmetry of the borehole and U-tube assembly so that only half of each pipe of the U-tube is modeled (see Figure 3). The outer extent of the domain has been selected so that

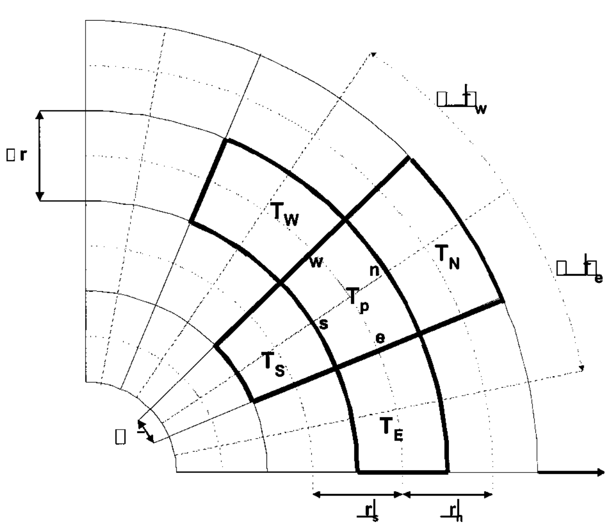

Figure 2 *Notation of the finite control volumes.*

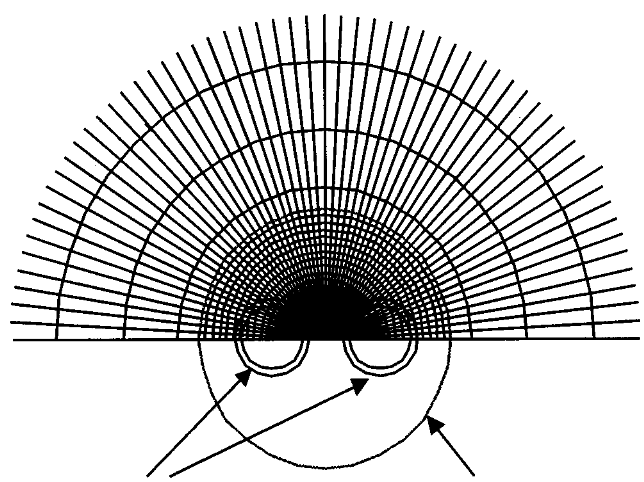

Figure 3 *Example numerical grid near the borehole center.*

it is large enough to approximate that of an infinite medium within the time scale of the calculations, i.e., the boundary temperature does not rise above the initial condition or the boundary heat flux change from zero. Initial tests with calculations up to 200 hours indicated that an outer radius of 12 ft (3.6 m) was sufficiently large.

Temperature gradients nearest the pipes are generally the steepest—particularly when the short time scale thermal response is considered. The grid is accordingly denser in this region, with the actual grid spacing being largely determined by the need to accurately represent the pipe geometry as discussed below. Beyond the borehole, the grid spacing is gradually expanded in the *r* direction to the domain boundary. The domain in the radial direction is divided into approximately 100 to 250 finite volume cells.

Even though it is straightforward to represent other borehole elements on the cylindrical coordinate system, care needs to be taken in the representation of the circular U-tube legs in the numerical domain. The polar coordinate system does not allow for a direct representation of a circular element that is offset from the center of the domain. In the grid generation scheme used here, a ''pie-sector'' approximation of the circular pipe geometry is implemented, as shown in Figure 4.

The parametric grid generation algorithm determines the size and position of the pie sector along with the thickness and number of the cells representing the pipe wall. Referring to Figure 4, the geometry of the pie sector is determined as follows. The position of the pie sector within the grid (point A′) is firstly determined by the shank spacing, which is one of the input parameters. Since the pipe wall thickness is also a parameter, the inside and the outside wall radii for the pie sector on the domain can be determined (points A′ and B′). Between point D and C the cells representing the pipe vary in thickness and so the thickness at the midpoint of D–C has to be fixed at the pipe thickness. The inside radial distance of

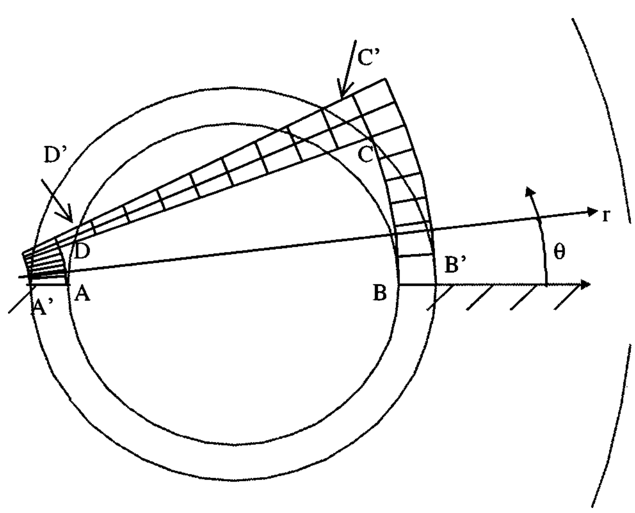

Figure 4 *The pie sector representation of the U-tube pipe.*

the pie sector (distance A–B) is fixed to correspond to the inside diameter of the pipe.

In order to approximate the heat flux boundary conditions at the pipe wall in the pie sector geometry, the inside perimeters have to be made equivalent. This is achieved by adjusting the distance between points B–C–D–A. A fixed angular grid spacing $\Delta\theta$ is used throughout the whole domain, so that the distances A–D and B–C are functions of the number of cells between these points and the total number of cells in the θ direction. These are determined by an iterative process until the perimeter of the pie sector closely matches, but is slightly greater than, the pipe perimeter. With approximately 200 cells in the radial and 100 in the θ direction the perimeter can be matched to within 0.05%.

A further consideration, given the implementation of the boundary conditions via source terms, is the number of cells that are used to represent the pipe wall. A range of values has been used and the corresponding sensitivity of the results is discussed below.

BOUNDARY CONDITIONS

The fluid in the pipes of the U-tube is not explicitly modeled and so the heat transfer from the fluid is treated by a heat flux boundary condition at the pipe walls. This heat flux is fixed for the unit depth of borehole modeled but some attempt has been made to account for the variation of temperature between the flow and return legs of the U-tube. This has been done by assigning 60% of the total heat flux to one leg and 40% to the adjacent leg. Although this heat transfer distribution is somewhat arbitrary, a sensitivity analysis showed only insignificant differences in average borehole temperature predictions when the distribution was varied between the 0%-100%–case and the 50%-50%–case.

At the start of the simulation a constant far-field temperature (normally equal to the undisturbed ground temperature) is assumed to be effective over the entire domain. The physical domain is represented as a semicircular grid by making use of the symmetry of the borehole geometry. The conditions at the symmetry plane are equivalent to a zero heat flux condition. At the outer edge of the domain (in the radial axis) a constant far-field temperature condition is applied. The amount of heat flux at the outer edge of the domain is continuously checked and verified to be zero or insignificantly small.

On the boundary surface of each of the finite control volumes along the perimeter of the pie sector, a constant heat flux term is applied entering/exiting the numerical domain. The amount of this flux is matched with a constant flux that would be transferred through a unit depth of pipe. As the code used here (Patankar 1991) has no convenient way of applying wall heat flux boundary conditions at cells interior to the domain, the boundary conditions at the pipe inside surface have been implemented via source terms in the cells comprising the inside of the pipe wall. Since the volume of each control volume changes along the perimeter of the pie sector, the source terms applied in these cells vary for a given boundary heat flux so that

$$S_C = \frac{q_B A_{i,j}}{\Delta V_{i,j}}, \tag{10}$$

where S_C is the source term, $q_{B_{i,j}}$ is the boundary flux, $A_{i,j}$ is the boundary surface area, and $\Delta V_{i,j}$ is the volume of the corresponding cell.

In order to be able to simulate the effects of the convective resistance due to fluid flow inside the U-tube pipe, the pipe conductivity in the numerical model has been adjusted so that the unit thermal resistance is the same as that of the pipe. The convective heat transfer coefficient was calculated using the Dittus-Boelter correlation (Incropera and DeWitt 1996) after averaging the exponent of the Prandtl number to arrive at a single value for both the heat extraction and rejection cases, so that

$$h_{in} = \frac{(0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.35}) k_{Fluid}}{D_{in}}. \tag{11}$$

VALIDATION OF THE NUMERICAL MODEL

Exact analytical solutions of the two-dimensional conduction heat transfer problem in a U-tube/borehole geometry, or a pie sector representation of it, are not known to exist. In view of this, some validation of the numerical method has been attempted using a related problem with known analytical solutions in one dimension. In seeking to validate the numerical method, the first aim has been to examine the significance of representing the pipes of the U-tube by a pie sector arrangement of cells. The second has been to investigate the sensitivity of the solution to grid density and time step size.

The most comparable analytical case to that modeled numerically is that of transient conduction through an infinitely long hollow cylinder (with the inside and outside cylindrical surfaces coaxial). The numerical method was initially tested for this simpler coaxial geometry and found to give very small errors compared to the analytical solution. The results presented here, however, compare the numerical results using a pie sector representation of a single pipe with this analytical solution, so that the effect of the simplified pipe geometry can be shown. In these tests the numerical domain outside of the pipe was also given a single value of thermal conductivity (i.e., grout material was given the same thermal properties as the soil). The boundary conditions were set to be a constant heat flux at the inside surface of the hollow cylinder at $r = r_{in}$ and a constant far-field temperature (equal to the initial temperature of the entire domain) at the outside surface at $r = r_{out}$. The implementation of these boundary conditions yields the following analytical solution for the temperature as a function of time and the radial coordinate (Carslaw and Jaeger 1947).

$$T(r,t) = \frac{r_{in} q_c}{k} \log\left(\frac{r_{out}}{r}\right) + \frac{\pi q_c}{k} \sum_{n=1}^{\infty} e^{-\alpha \beta_n^2 t} \times \frac{J_0^2(r_{out}\beta_n)[J_0(r\beta_n)Y_1(r_{in}\beta_n) - Y_0(r\beta_n)J_1(r_{in}\beta_n)]}{\beta_n[J_1^2(r_{in}\beta_n) - J_0^2(r_{out}\beta_n)]}, \tag{12}$$

where β_n's are the positive roots of

$$J_1(r_{in}\beta)Y_0(r_{out}\beta) - Y_1(r_{in}\beta)J_0(r_{out}\beta) = 0. \tag{13}$$

Comparisons have been made between the time varying temperatures at the inside surface of the cylinder with the numerical result being given by an area weighted average of the inside surface temperature of the pie-sector cells. In order to make a meaningful comparison between the analytical solution and the numerical results, a modification has to be made to the temperature given by the above analytical solution to account for the additional resistance at the inside pipe wall which is accounted for in the numerical model. This has been done by calculating an increase in resistance that would normally apply in steady state conditions. This is given in terms of an adjustment to the inside pipe surface temperature given by the analytical solution as follows:

$$\Delta T_{adj} = \frac{Q_c}{2\pi r_{in} L} \left\{ \frac{r_{in} \ln(r_{out}/r_{in})}{k_{pipe}} + \frac{1}{h_{in}} \right\}. \tag{14}$$

Although some error is introduced by assuming this resistance is the same as in steady state conditions, this error diminishes rapidly with time.

Model Validation Test Cases

Six different test cases were established for making the comparisons between the numerical results and the analytical solutions. These used a range of different pipe diameters, far field temperatures, heat fluxes and thermal conductivities. The borehole geometry and material thermal properties have been selected to include common values used in vertical ground loop heat exchangers. In the test cases the applied heat fluxes varied between 212 Btu/hr-ft^2 (668.6 W/m^2) and 135 Btu/hr-ft^2 (425.7 W/m^2). The parameters varied between each of the test cases are given in Table 1. The remaining parameters common to all test cases are given in Table 2. All of the test case calculations were run for a simulated time of 192 hours.

Sensitivity to Grid Resolution and Time Step

Given the simplified way in which the heat flux boundary condition had to be applied at the cells representing the inside surface of the pipe—and the importance of the temperature gradient prediction in this region to the short time scale response—some effort was made to investigate the grid independence of the numerical solution. This was done by making calculations with 1, 2, 4, or 8 cells representing the thickness of the pipe wall. (As the number of cells in the

TABLE 1
Input Data Varied for Model Validation Test Cases

Property	Test Case					
	1	2	3	4	5	6
Q Btu/hr-ft^2 (W/m^2)	211.9 (668.5)	211.9 (668.5)	169.7 (535.4)	169.7 (535.4)	134.4 (423.9)	134.4 (423.9)
$T_{farfield}$ F (C)	63.0 (17.2)	48.0 (8.9)	63.0 (17.2)	48.0 (8.9)	63.0 (17.2)	48.0 (8.9)
k_{soil} Btu/hr-F-ft (W/m-K)	1.5 (2.6)	1.0 (1.73)	1.5 (2.6)	1.0 (1.73)	1.5 (2.6)	1.0 (1.73)
k_{grout} Btu/hr-F-ft (W/m-K)	1.5 (2.6)	1.0 (1.73)	1.5 (2.6)	1.0 (1.73)	1.5 (2.6)	1.0 (1.73)
$D'_{PipeOuter}$ ft. (mm)	0.0875 (26.5)	0.0875 (26.5)	0.1096 (33.4)	0.1096 (33.4)	0.1383 (42.2)	0.1383 (42.2)
$d_{PipeWall}$ ft. (mm)	0.0079 (2.4)	0.0079 (2.4)	0.01 (3.0)	0.01 (3.0)	0.013 (3.9)	0.013 (3.9)

pipe wall was increased in the radial direction, some increase in the number of cells in the angular direction was also made to ensure the cell aspect ratio was not excessive.) The temperature predictions have been compared in terms of relative error where the error is scaled according to the difference between the analytically determined surface temperature and the far field (initial) temperature at each time step so that,

$$Relative\ \ Error = \frac{\lfloor T_{Analytical} - T_{Predicted} \rfloor}{\lfloor T_{Analytical} - T_{FarField} \rfloor} \cdot 100(\%). \tag{15}$$

The resulting relative errors found after one hour and 192 hours of simulation time are given in Table 3. The distribution of the relative error with time for different grid resolutions at the pipe wall is given in Figure 5. It can be seen that there is a noticeable difference between the results with one and two cells representing the pipe wall. There is further improvement in agreement with the analytical temperatures as the grid density is increased to four and eight cells. However, there is a smaller difference between the cases with four and eight cells. From these results, four cells representing the thickness of the pipe wall was selected as the most appropriate number to simulate the pipe geometry with the pie-sector approximation. The relative error for this case is consistently less than 1% after 192 hours of simulation time (Table 3) and further increase in the grid resolution does not appear to justify the increased computational cost.

The first order fully implicit backward differencing in time approach taken here has the advantage that the numerical solution is stable with a wide range of time step size. This differencing scheme, however, gives only first order accuracy in time and some variation in accuracy with time step size could be expected. In view of this, calculations were made using the parameters of test case number 4 and with time steps in the range 0.1–10 minutes. The resulting variation in the relative error of the temperature prediction using this range of time steps is shown for the first 24 hours of the calculation in Figure 6. The relative errors can be seen to decrease with smaller time steps and are generally greatest near the beginning of the calculation. This later feature can be expected as under these boundary conditions the rate of temperature change decreases with time. As the pipe surface temperature approaches the steady-state value, the relative influence of the time step on the temperature predictions therefore becomes smaller. Again there is a point of diminishing returns in selecting a shorter time step size in light of the increased computing times. Three-minute time steps were chosen for use in further analysis on the basis of being a reasonable compromise between prediction accuracy and computational speed.

The parametric grid generation scheme used here is intended to deal with U-tubes with a range of shank spacings. (Shank spacing is defined here as the size of the gap between the pipes of the U-tube.) One feature resulting from the representation of the pipe by a pie sector is that its exact shape in the grid varies depending on the shank spacing (i.e., distance

TABLE 2
Input Data Common to All Validation Test Cases

k_{pipe} **Btu/hr-F-ft (W/mK)**	**0.226 (0.391)**
$(\rho c)_{pipe}$ Btu/ft^3-F (kJ/m^3-K)	30.0 (2,012.1)
$(\rho c)_{soil}$ Btu/ft^3-F (kJ/m^3-K)	35.0 (2,347.5)
$(\rho c)_{grout}$ Btu/ft^3-F (kJ/m^3-K)	35.0 (2,347.5)
$r_{borehole}$ ft (mm)	0.1458 (44.4)

TABLE 3
Relative Error (%) Between the Analytical and Numerical Results for Each Test Case at 1 and 192 Hours Simulated Time

	Case #1		Case #2		Case #3		Case #4		Case #5		Case #6	
Time (hr)	1	192	1	192	1	192	1	192	1	192	1	192
# of cells												
1	−4.48	−2.51	−3.24	−1.58	−5.39	−3.14	−3.80	−2.11	−1.95	−0.32	−1.43	−0.11
2	−0.62	−0.48	−0.08	−0.93	−1.70	−1.63	−0.48	−0.83	−2.07	−1.75	−0.80	−0.88
4	2.91	−0.49	3.44	−0.02	2.62	−0.57	3.17	−0.06	1.85	−0.62	2.65	−0.13
8	11.15	1.29	8.88	0.32	8.59	0.03	8.49	0.75	8.28	0.69	8.06	0.43

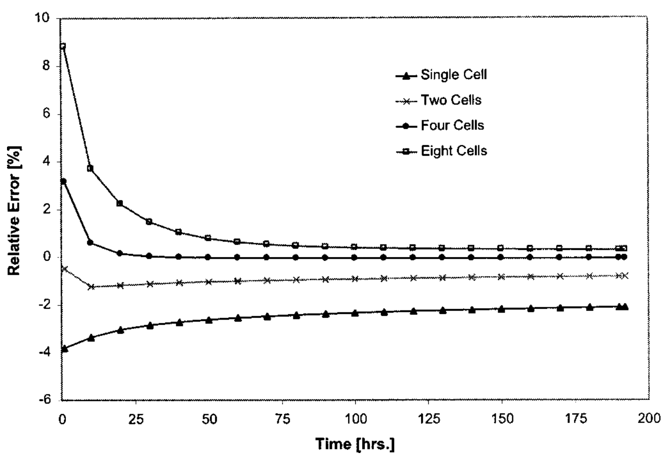

Figure 5 *Variation of the relative error of the calculated temperature with different grid resolution at the pipe wall (for test case #4 with 3-minute time steps).*

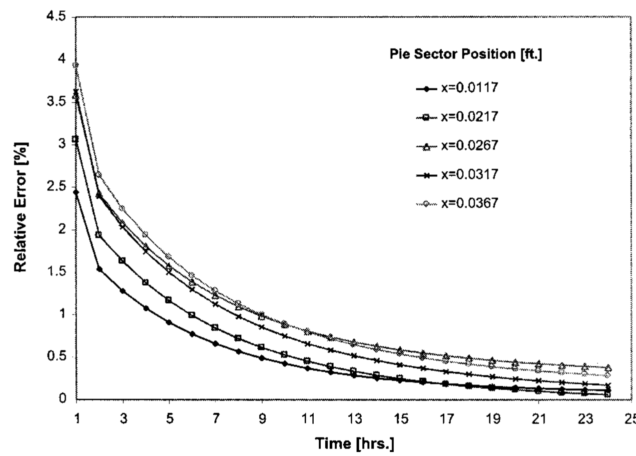

Figure 7 *Variation in the relative error of the calculated temperature with pie-sector position (test case #4).*

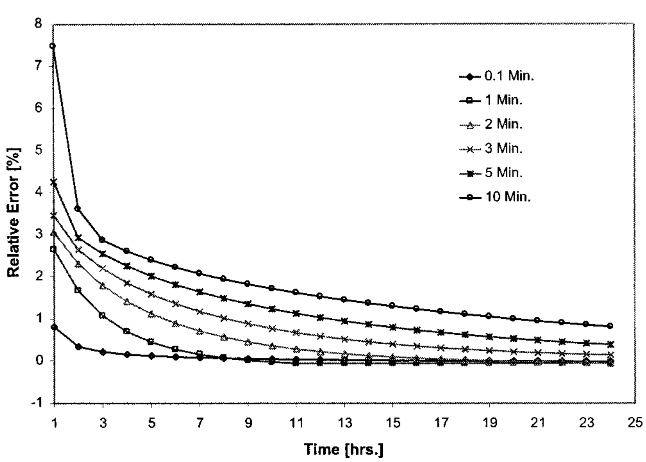

Figure 6 *Variation in the relative error of the calculated temperature with time step size (test case #4).*

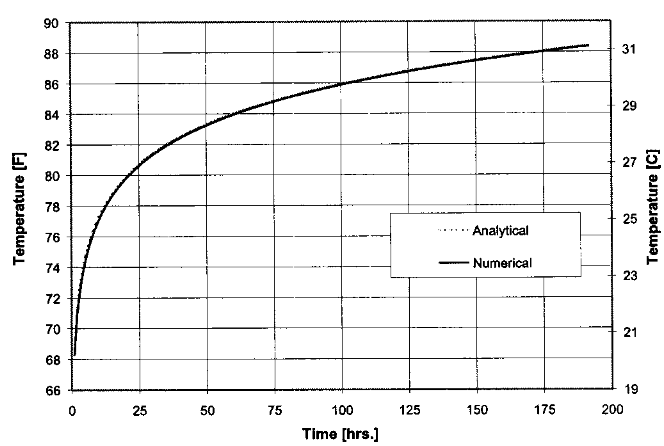

Figure 8 *Comparison of the numerical and analytical model temperature predictions. Test case #4. Pipe wall thickness approximated with 4 cells. Time step=3 minutes.*

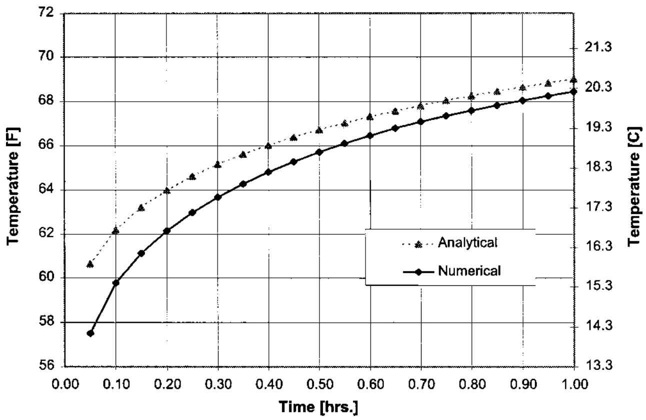

Figure 9 *Comparison of the numerical and analytical model temperature predictions over the first hour of simulation (test case #4).*

from the origin). This arises because in order to match the perimeter of the pipe with that of the pie sector the included angle of the pie sector has to increase if it is positioned nearer the origin. When comparing the calculated temperature for a single pipe with those of the analytical model, as here, one would ideally want the predictions to be insensitive to the position of the pipe in the numerical domain. (Where there are two pipes with heat fluxes applied, however, there may be some real sensitivity to shank spacing.)

To examine the sensitivity of the results to the position of the pie sector a number of calculations were made with the position from the grid origin varying from 0.012 ft (3.6 mm) to 0.037 ft (11.3 mm) and other parameters as test case number 4. The resulting error distribution for these calculations is shown in Figure 7. Again the errors appear largest near the beginning of the calculation. The smallest errors are given by the cases with the pie sector having the smaller offset from the origin. However, the relative error for each of these calculations is contained within a band of less than 1% after about 12 hours indicating that the sensitivity of the comparison with the analytical solution to pie sector position is only very slight.

The grid generation and time stepping practice finally adopted was to use four grid cells to represent the pipe wall thickness and to use time steps of 3 minutes. Figure 8 shows the final results for test case number 4 where the predicted inner surface temperature is shown against that given by the analytical model. In this case the relative error is about 3% after the first hour of simulation and decreases to a value of approximately 0.15% of the total temperature rise after the 24th hour, reducing to a final value of 0.06% after 192 hours. Figure 9 illustrates the same results plotted for the first hour. Even though some lag is observed in the predicted temperature over the first hour, the relative error remains small and diminishes thereafter. The absolute temperature error after the first three minutes is approximately 2.7°F (1.5°C) decreasing to 0.5°F (0.28°C) after 30 minutes. Similar behavior was observed in the other five test cases, where the average relative error was found to be smaller than 1% of the temperature rise.

CONCLUSIONS

A transient, two-dimensional finite volume numerical model has been developed for calculation of conduction heat transfer in and around the borehole of a vertical U-tube ground loop heat exchanger. A method has also been devised that allows the numerical grid for a range of borehole and U-tube configurations to be generated automatically from a small set of geometric parameters.

A range of calculations has been made to validate the results of the numerical model against the time varying temperatures given by a comparable analytical model. The grid generation practices and time step size have been refined so that the numerical model is capable of predicting the pipe surface temperature with an average relative error of ±1% compared to the analytically calculated temperature. The errors were found to be more significant in the first hours of simulation where the temperature differences were smallest and the rate of change of temperature greatest. The errors always diminished rapidly and became insignificant before the end of that 192-hour period considered here. These errors are considered acceptably small for the prediction of short time scale response of the heat exchanger for design and simulation purposes.

Although the discretization errors have been adequately minimized, and the pie sector representation of the pipe geometry in the polar coordinate system shown to be sufficient, this has required a notably dense computational grid (approximately 100 by 200 cells). Further benefit may be gained by using a boundary fitted grid system that would allow more accurate representation of the borehole geometry. Such an approach may also require a less dense grid for a given level of accuracy and therefore offer some decrease in computational load. This is the subject of further research.

The numerical model reported here has been developed for use in two applications that require the prediction of the short time scale response of the borehole. The first is to provide the thermal response of the heat exchanger on shorter time scales (up to one month) for heat exchanger design purposes and component-based simulation. This has been done by using the numerical model to derive nondimensional thermal response curves (Yavuzturk and Spitler 1999). The second application is modeling the short-term response under *in situ* conductivity test conditions. In this case the model has been used to solve the inverse heat transfer problem associated with estimating ground and grout thermal conductivity from *in situ* test data. These data are generated by applying a single heat pulse to a particular ground loop heat exchanger and using a numerical optimization technique to find an estimate of the ground and grout thermal conductivity (Austin 1998; Austin et al. 2000).

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NOMENCLATURE

a = conductance (Btu/hr.°F[W/°C])
A = area (ft^2[m^2])
C = specific heat (Btu/lb-F [kJ/kg-K])
D' = diameter (ft [m])
h = convective heat transfer coefficient (Btu/hr-ft^2-F[W/m^2-K])
J_0 = Bessel function of first kind zeroth order
J_1 = Bessel function of first kind first order
k = thermal conductivity (Btu/hr-ft-°F[W/m-°C])
L = borehole depth (ft [m])
Pr = Prandtl number
r = radius (ft [m])
Re = Reynolds number
q = heat flux (Btu/hr-ft^2[W/m^2])
Q = heat transfer rate (Btu/hr [W])
S = source term (Btu/hr-ft^3[W/m^3])
t = time (s)
T = temperature (°F[°C])
V = volume (ft^3[m^3])
Y_0 = Bessel function second kind of zeroth order
Y_1 = Bessel function second kind of first order

Symbols

α = thermal diffusivity (ft^2/hr[m^2/s])
β = integration variable in the analytical solution
ρ = density (lb/ft^3[kg/m^3])
θ = angular coordinate

Subscripts

adj = adjusted
B = boundary
e, E = east
i, j = cell index
n, N = north
P = current cell
s, S = south
w, W = west

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