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A Short Time Step Response Factor Model for Vertical Ground Loop Heat Exchangers

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ABSTRACT

The short-term behavior of ground-coupled heat pump systems is important for design of ground loop heat exchangers, energy analysis of ground source heat pump systems, and design of hybrid ground source systems. This paper describes the development of short time-step temperature response factors for vertical ground loop heat exchangers as used in ground-coupled heat pump systems. The short time-step response factors allow for a direct evaluation of system energy consumption and electrical demand in hourly or shorter time intervals. The development of the temperature response factors is based on an analytically validated, transient two-dimensional implicit finite volume model designed for the simulation of heat transfer over a vertical U-tube ground heat exchanger. The short time-step response factors are implemented as part of a component model for TRNSYS and an example application is provided based on an actual building.

INTRODUCTION

The advantages of ground-coupled heat pump systems over their conventional alternatives with respect to system life cycle costs, operating costs, and environmental friendliness make them a viable alternative for residential and commercial HVAC systems. Ground-coupled heat pump applications can make significant contributions to reductions in electrical energy usage and improvements in load profiles. However, despite the perceived economic benefits of such systems, there has been almost no work reported on detailed models suitable for hourly energy analysis.

In a ground-coupled heat pump application, the actual heat transfer to and from the ground loop heat exchanger varies continuously due to changing building energy requirements. These changes result in short time-step fluctuations in the supply and return temperatures of the ground loop and can typically vary up to 10–18°F (5.6–10°C) over a given day. The resulting variations have a direct impact on the coefficient of performance (COP) of the heat pump unit and thus influence the overall system performance in a significant way. In cases where time-of-day electricity rates are applicable, the impact of fluctuating performance on the system economics will be even more significant.

For a detailed building energy analysis, a ground loop heat exchanger simulation model is called for that can reliably and efficiently predict the short term fluctuations of the ground loop heat exchanger return temperature during a given day. This enables the determination of energy consumption and demand information on an hour-by-hour basis. In ground loop heat exchanger design programs, the actual daily load profile is often approximated as a single fixed load with a user-specified duration. A true hourly model of the ground loop heat exchanger can be used to eliminate this approximation.

In addition to building energy analysis and ground loop design applications, the short time-step model can be used for hybrid ground source applications. In some situations, for example, where cooling loads are very dominant, supplemental heat rejecters such as cooling towers are used. Various operating strategies might be utilized in hybrid systems, for example, to reduce heat buildup in the ground by running during the winter or by running at night during the summer. In order to quantify the impact of various operating strategies on ground loop heat exchanger size and operating costs, a model that can account for changes in the hourly load profile and interaction between the ground loop heat

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exchanger and heat rejecter is needed.

A review of the literature on the currently available system design and simulation tools for vertical ground loop heat exchangers yields a multitude of design approaches that range from crude rule-of-thumb approximations to detailed analytical and/or numerical techniques. However, none of the approaches at their current level of sophistication is capable of modeling and simulating multiple-borehole ground loop heat exchangers in short time intervals (hourly or shorter) with the necessary accuracy. Most of the design and simulation programs require monthly building loads and provide monthly average ground loop entering and exiting temperatures of the heat transfer fluid. Some models take a slightly more detailed approach by requiring the input of peak loads. This allows for the calculation of peak loop entering and exiting temperatures during a month, but determining exactly when they occur during the given month is not possible.

A good number of the analytical design approaches are based on Kelvin's (1882) line source theory or its derivations by Ingersoll et al. (1954). The line source approach approximates the ground loop borehole with the U-tube pipe as an infinitely long line with radial heat flow. The short time-step system behavior cannot be modeled directly since the approach is exact only for a true line source and can be applied to cylindrical heat sources with acceptable error only after several hours of system operation. According to Ingersoll et al., the criterion for the error term is that the dimensionless ratio of the product of soil diffusivity and time to the source radius ($\alpha t/R^2$) be greater than 20. For typical applications with ordinary soils the smallest resulting time is about 12 hours at which, according to Ingersoll et al. (1954), the error does not exceed about 2%.

The cylinder source method as developed by Carslaw and Jaeger (1947) and derivative methods such as that of Deerman and Kavanaugh (1991) are widely used and considered to be more accurate than the line source approach. In the cylinder source models, an analytical solution is developed for a region bounded internally by a cylinder of a constant radius. For zero initial temperature, a constant radial heat flux per unit time and per unit area is stipulated at the radius of the cylinder. A simplifying assumption that treats both legs of the U-tube as a single cylinder with an equivalent diameter introduces significant errors in determining system short time-step behavior.

An even higher level of detail and sophistication in system design and simulation can be obtained through detailed numerical approaches that model individual borehole elements (U-tube, grout, etc.) by physically defining them within the numerical domain of interest—a single borehole and its immediate surroundings. This is usually done on a two-dimensional polar grid. A number of different stand-alone numerical models (Mei 1985; Muraya et al. 1996; Rottmayer et al. 1997; Yavuzturk et al. 1999) have been developed to describe the heat transfer process in and around the vertical ground heat exchanger borehole based on either finite difference or finite element equations with varying model sophistication and accuracy. Nevertheless, numerical models using polar or cylindrical grids cannot model multiple borehole configurations and the computational resources to obtain the time-varying average borehole field temperature on the more complex grids necessary would be prohibitively high.

Hellstrom (1989, 1991) developed a simulation model for vertical ground heat stores, which are densely packed ground loop heat exchangers used for seasonal thermal energy storage. The model represents the total change in the initial ground temperature for a time step first by the spatial superposition of three parts: a so-called ''global'' temperature difference due to heat conduction between the bulk of the heat store volume with multiple boreholes and the far field, a temperature difference from the ''local'' solution immediately around the heat store volume, and a temperature difference from the ''local'' steady-flux part. The average ground temperature at any subsequent time is determined by decomposing the time-varying heat transfer profile into a series of individual step heat pulses and then superimposing the resulting responses in time. The model is a hybrid model that uses a numerical solution for the ''local'' and the ''global'' problems and then superimposes them spatially with the analytical solution from the steady-flux part. The numerical model uses a two-dimensional explicit finite difference scheme on the radial-axial coordinate system for the ''global'' problem. For the local solution, a one-dimensional radial mesh is used that divides the storage region into several subareas. Hellstrom's model is not ideal for determining long time-step system responses for ground source heat pump systems since the geometry of the borehole field is assumed to be densely packed, with a minimum surface area to volume ratio, as is typical for heat stores.

Thornton et al. (1997) used Hellstrom's approach as part of a detailed component-based simulation model of a ground source heat pump system. The model was implemented in TRNSYS (Klein et al. 1996). It was calibrated to monitored data from a family housing unit by adjusting input parameters such as the far-field temperature and the soil thermal properties. When calibrated, the model was able to accurately match measured entering water temperatures.

The objective of this paper is to develop nondimensional short time-step temperature response factors in vertically ground-coupled heat exchangers as an extension of the system design and simulation method based on long time-step response factors as provided by Eskilson (1987). The development of the short time-step response factors are based on an analytically validated, transient two-dimensional implicit finite volume model that simulates the heat transfer over a vertical U-tube ground heat exchanger as developed by Yavuzturk et al. (1999). In order to provide an example for its application as a tool for the short time-step building energy analysis, the model is cast as a component model for TRNSYS (Klein et al. 1996) simulating a real building located in Tulsa, Oklahoma. The results of this simulation are reported below.

TEMPERATURE RESPONSE FACTORS

Long Time-Step Temperature Response Factors

Eskilson's (1987) approach to the problem of determining the temperature distribution around a borehole is a hybrid model combining analytical and numerical solution techniques. A two-dimensional numerical calculation is made using the transient finite-difference equations on the radial-axial coordinate system for a single borehole in homogeneous ground with constant initial and boundary conditions. The capacitance of the individual borehole elements such as the pipe wall and the grout are neglected. The solution obtained using a basic step pulse allows the calculation of response to any heat input by considering piecewise constant heat extractions/rejections and superpositioning them in time as a series of step pulses.

The temperature response of the borehole from the discretized equations is converted to a series of nondimensional temperature response factors. (Eskilson calls these response factors *g* functions. They should not be confused with *g* functions used in the cylinder source solution.) The *g* function allows the calculation of the temperature change at the borehole wall in response to a step heat input for a time step. Once the response of the borehole field to a single step heat pulse is represented with a *g* function, the response to any arbitrary heat rejection/extraction function can be determined by devolving the heat rejection/extraction into a series of step functions, and superimposing the response to each step function.

This process is graphically demonstrated in Figure 1 for four months of heat rejection. The basic heat pulse from zero to $Q1$ is applied for the entire duration of the four months and is effective as $Q1' = Q1$. The subsequent pulses are superimposed as $Q2' = Q2 - Q1$ effective for three months, $Q3' = Q3 - Q2$ effective for two months and finally $Q4' = Q4 - Q3$ effective for one month. Thus, the borehole wall temperature at any time can be determined by adding the responses of the four step functions. Mathematically, the superposition gives the borehole temperature at the end of the nth time period as

$$T_{borehole} = T_{ground} + \sum_{i=1}^{n} \frac{(Q_i - Q_{i-1})}{2\pi k} g\left(\frac{t_n - t_{i-1}}{t_s}, \frac{r_b}{H}\right) \quad (1)$$

where:

t = time (s),

t_s = time scale = $H^2/9\alpha$,

H = borehole depth, ft (m),

k = ground thermal conductivity, Btu/hr-ft-°F (W/m-°C),

$T_{borehole}$ = average borehole temperature in °F (°C),

T_{ground} = undisturbed ground temperature in °F (°C),

Q = step heat rejection pulse, Btu/hr-ft (W/m),

r_b = borehole radius, ft (m),

i = index to denote the end of a time step (the end of the first hour or second month, etc.)

Figure 2 shows the temperature response factor curves (*g* functions) plotted versus nondimensional time for various multiple borehole configurations and compares them to the temperature response factor curve for a single borehole. The *g* functions in Figure 2 correspond to borehole configurations with a fixed ratio of 0.1 between the borehole spacing and the borehole depth. The thermal interact ion between the boreholes is stronger as the number of boreholes in the field is increased. The interaction increases as time of operation increases.

The detailed numerical model used in developing the long time-step *g* functions approximates the borehole as a

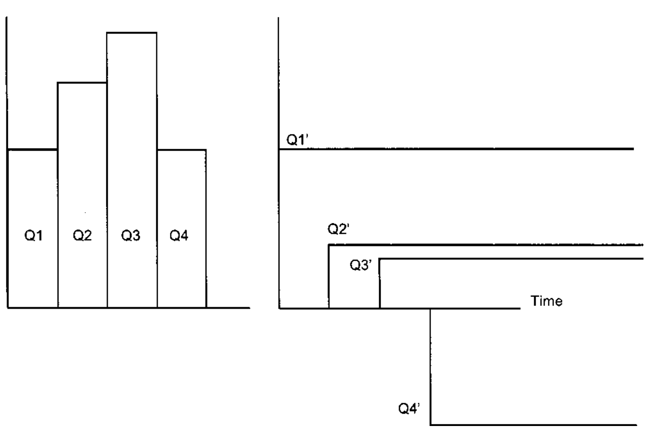

Figure 1 *Superposition of piecewise linear step heat inputs in time. The step heat inputs $Q2$, $Q3$, and $Q4$ are superimposed in time on to the basic heat pulse $Q1$.*

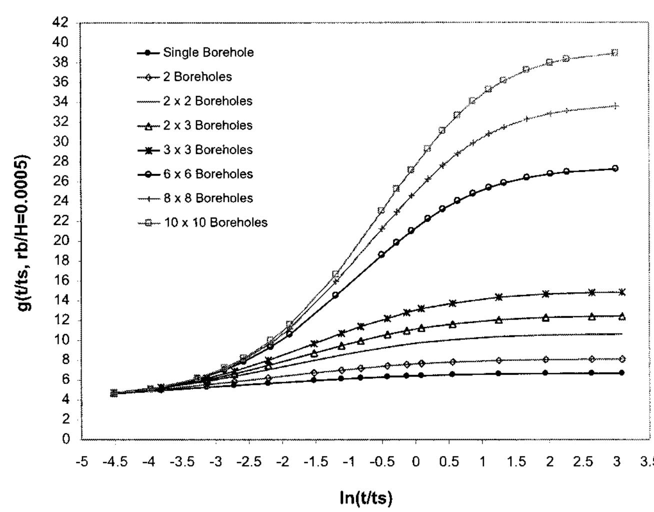

Figure 2 *Temperature response factors (g functions) for various multiple borehole configurations compared to the temperature response curve for a single borehole.*

line source of finite length, so that the borehole end effects can be considered. The approximation has the resultant problem that it is only valid for times estimated by Eskilson, to be greater than $5r_{Borehole}^2/\alpha$. For a typical borehole, that might imply times from 3 to 6 hours. However, for the short time step model, it is highly desirable that the solution be accurate down to an hour and below. Furthermore, much of the data developed by Eskilson does not cover time periods of less than a month. [For a heavy, saturated soil and a 250 ft (76.2 m) deep borehole, the g function for the single borehole presented in Figure 2 is only applicable to about 60 days.]

Development of Short Time-Step Temperature Response Factors

The approach to extending the current long time-step model to short time steps involves the use of a detailed numerical model to develop g functions that are suitable for shorter time periods (hourly or less) than currently available. The numerical model is used to determine the time-dependent average borehole temperatures for a step pulse of a given borehole geometry and known ground thermal conductivity. The temperature response is nondimensionalized to form g function values.

Two-Dimensional Numerical Borehole Model

The numerical model that is used to compute short time-step average borehole temperatures uses a transient two-dimensional implicit finite volume discretization on a polar grid. It was developed to simulate the heat transfer over a vertical U-tube ground heat exchanger of a ground-source heat pump system and has been validated by comparison to applicable analytical solutions (Yavuzturk et al. 1999).

A sketch of the numerical domain is provided in Figure 3. Since there is a symmetry axis through the borehole, only one half of the borehole is modeled. For a typical borehole, a grid resolution of about 100 finite volume cells in the angular direction and about 150 to 200 cells in the radial direction is utilized. The exact grid resolution is a function of the borehole and U-tube pipe geometry and is determined by an automated parametric grid generation algorithm. The radius of the numerical domain is 12 ft (3.6 m) to allow for reasonably long simulation times.

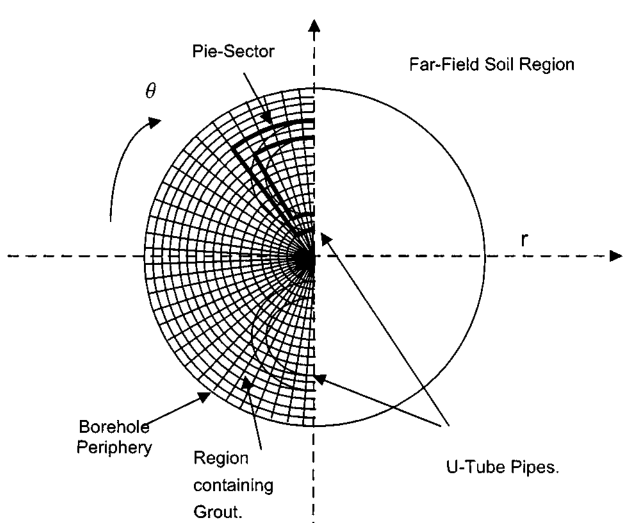

Figure 3 *Simplified representation of the borehole region on the numerical model domain using the pie-sector approximation for the U-tube pipes.*

The geometry of the circular U-tube pipes is approximated by so-called pie sectors over which a constant flux is assumed to be entering the numerical domain for each time step. The pie-sector approximation attempts to simulate the heat transfer conditions through a circular pipe by matching the inside perimeter of the circular pipe to the inside perimeter of the pie sector and by establishing identical heat flux and resistance conditions near the pipe walls. The convection resistance due to the heat transfer fluid flow inside the U-tubes is accounted for through an adjustment on the conductivity of the pipe wall material.

The initial condition of the numerical model stipulates a constant, undisturbed domain temperature corresponding to the far field temperature. Due to the symmetry in the numerical domain a zero heat flux condition is implemented in the angular direction while the heat transfer from/to the U-tube pipes (the pie sectors that model the U-tube pipes) are input as fixed boundary flux conditions. Since the total amount of boundary heat flux over each U-tube pipe is not the same, a 60% vs. 40% heat transfer distribution over the pipes of the U-tube is assumed. Although this heat transfer distribution is somewhat arbitrary, a sensitivity analysis showed only insignificant differences in average borehole temperature predictions when the distribution was varied between the 0%-100%–case and the 50%-50%–case. Finally, the boundary condition in the radial axis is set to be the constant far field temperature. The simulation time step is 2.5 minutes.

Borehole Resistance

The numerical approach that is used to develop the short time-step g functions also models the thermal effects of the individual borehole elements such as the resistance of the pipe and grout material due to heat conduction and the convection resistance due to the flow of the heat transfer fluid inside the U-tube pipes. Since the g function values as developed by Eskilson (Figure 2) do not include these thermal resistance effects, the short time-step g function values need to be adjusted accordingly. The following relationships are used for the grout, U-tube pipes, and the convection resistance per unit borehole length:

$$R_{Grout} = \frac{1}{k_{Grout}\beta_0(D_{Borehole}/D_{Pipe})^{\beta_1}}, \tag{2}$$

$$R_{Convection} = \frac{1}{2\pi D_{in}h_{in}}, \tag{3}$$

$$R_{PipeConduction} = \frac{\ln(D_{out}/D_{in})}{4\pi k_{Pipe}}, \quad (4)$$

$$R_{Total} = R_{Grout} + R_{Convection} + R_{PipeConduction}, \quad (5)$$

where β_0, β_1 = Resistance shape factor coefficients (Paul 1996) based on U-tube shank spacing. Paul's (1996) shape factor coefficients are based on experimental and finite element analysis of typical borehole and pipe geometry. Shape factor coefficients of $\beta_0 = 20.100377$ and $\beta_1 = -0.94467$ are suggested for a typical 0.125″ (3.2 mm) U-tube shank spacing,

R = thermal resistance in F per Btu/hr-ft (in C per W/m),

D = diameter in ft (m),

k = thermal conductivity in Btu/hr-ft-F (W/m-C),

h_{in} = convection coefficient based on the inside diameter in hr-ft^2-F/Btu (m^2-C/W).

The convection coefficient is determined with the Dittus-Boelter correlation

$$h_{in} \cong \frac{0.023\,\mathrm{Re}^{0.8}\mathrm{Pr}^n k_{Fluid}}{2r_{in}}, \quad (6)$$

where $n = 0.4$ for heating and $n = 0.3$ for cooling; a mean value of 0.35 is used.

The total borehole resistance in F/BTU/hr-ft (C/W/m) for each time step is multiplied by the heat transfer rate per unit length of borehole for that time step to calculate the temperature rise adjustment. This temperature rise due to the total borehole resistance needs to be subtracted from the temperature value obtained through the numerical model to determine the actual temperature rise for that time step. Consequently, Equation 1 is recast to solve for the g function with a single step pulse and modified to account for the borehole thermal resistance:

$$g\left(\frac{t_i}{t_s}, \frac{r_b}{H}\right) = \frac{2\pi k\{T_{borehole} - (R_{Total}Q) - T_{ground}\}}{Q}. \quad (7)$$

The resulting short time-step g function values are plotted in Figure 4 side by side with the long time-step g function values for a single borehole and an 8×8-borehole field as given by Hellstrom (1998).[1]

The independently generated short time-step g functions line up very well with Eskilson's long time-step g functions. They are stored as a series of points $[g(t/t_s, r_b/H), \ln(t/t_s)]$ and are appended to the set of long time-step g functions. For typical ratios of borehole radius to borehole depth, the short time-step g function data correspond to time steps between 2.5 minutes and 200 hours. Although some overlapping between the long and short time-step g functions can be seen in Figure 4, the long time-step g functions are basically applicable for times longer than 200 hours. In use, g-function values are determined by linear interpolation between the nearest points.

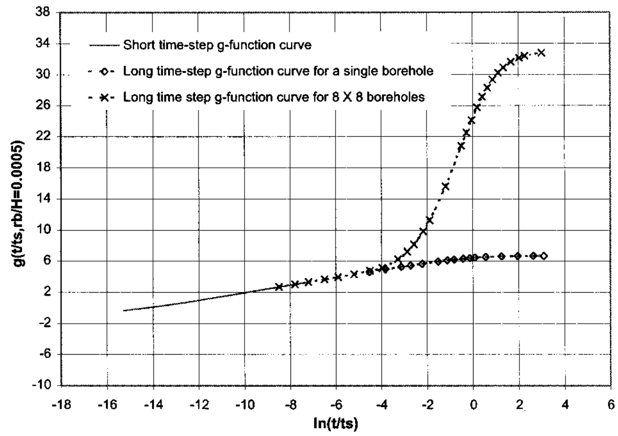

Figure 4 *Short time-step g function curve as an extension of the long time-step g functions plotted for a single borehole and an 8×8 borehole field.*

Aggregation of Ground Loads

The short time-step g functions developed thus far can be implemented in an algorithm to predict short time-step loop temperature variations. However, this would require that short time-step ground loads be devolved into individual step pulses and be superimposed in time for each time-step using the corresponding short time-step g function. Since the number of superposition calculations is proportional to the square of the number of time steps, an 8760-hour annual simulation creates a significant computational burden. Such an algorithm has been developed within the framework of this study, but it is a computationally inefficient way of determining short-term temperature variations on the ground loop heat exchanger.

In order to be able to reduce the computational time, an aggregation algorithm is developed for the ground loads considering that the importance of a load at a given time step diminishes for subsequent time steps as time progresses. That is, loads that occur more than a certain time ago can be ''lumped'' together into larger blocks. Thus the history of a multitude of short time-step loads can be represented in single load ''blocks'' to be superimposed onto more recent loads.

The aggregation algorithm keeps track of all hourly ground loads up to the current time step. An average ground load is then computed for user-definable ''blocks'' of time (for example, if the ground loads are given in hourly time steps then 730 hours worth of hourly loads may be averaged over this time period to represent one aggregate load for the 730-hour time block).

For the computation of temperatures inside the very first user-defined time block no load aggregation is performed. The load aggregation begins once the first time block has

[1]Figure 4 provides the g functions for an 8×8 borehole field as extended by Hellstrom. Hellstrom appended the original g functions of Eskilson using the line source approach for shorter time steps, thus allowing for time steps down to about 100 hours.

passed. (Using the earlier example, this means that the load aggregation would only start after the 730th hour of the simulation. Average borehole temperatures for earlier hours would be computed without any load aggregation with the short time-step g functions). For any given time-step after the first load-aggregated time block, the average borehole temperature is computed by first superimposing the aggregated loads from the 730-hour blocks and then by superimposing the short time-step loads upon the aggregated longer time-step block loads. Figure 5 shows a comparison of temperature responses based on hourly loads with temperature responses obtained through load aggregation for the first week and the first month for a typical case. As expected, the longer time blocks are averaged, the greater is the deviation from the actual hourly temperatures when the aggregation routine switches back to the hourly simulations.

To provide an example for the load aggregation consider a set of hourly load data where the computation of the average borehole temperature for the 2281st hour is sought. Defining the load aggregation time block to be 730 hours, the hourly short time-step loads can be aggregated for three larger time blocks. The average load values for each of these larger time blocks are determined by summing the hourly loads and then dividing the sum by 730 hours. The superpositioning of these three sets of aggregated loads with the corresponding longer time-step g functions (in this example g function values for 730, 1460, and 2190 hour duration) yields the temperature response at the end of the 2190th hour of simulation. Since the average borehole temperature at the end of the 2281st hour is sought, the hourly loads for the remaining 91 hours are then superimposed in hourly steps with the corresponding hourly g function values to obtain the temperature at the 2281st hour,

$$T_{2281}=T_{ground}+\sum_{m=1}^{3}\left[\frac{(\overline{q}_m-\overline{q}_{m-1})}{2\pi k_{ground}}g\left(\frac{t_{2281}-t_{(730m-730)}}{t_s},\frac{r_b}{H}\right)\right]+\sum_{n=2191}^{2281}\left[\frac{(q_n-q_{n-1})}{2\pi k_{ground}}g\left(\frac{t_{2281}-t_{n-1}}{t_s},\frac{r_b}{H}\right)\right] \quad (8)$$

where

m = index for the load aggregated time blocks,

n = index for the hourly time steps,

$\overline{q}$ = average aggregated load in Btu/hr-ft [W/m],

q = hourly load in Btu/hr-ft [W/m],

T_{2281} = temperature at the end of the 2281st hour,

t = time.

Note that $q_{2190}=\overline{q_3}$

The direct superposition of temperature responses based on aggregated loads with temperature responses based on short time-steps introduces an error in the final temperature as illustrated in Figure 6. The load aggregation block for this example is assumed to be 730 hours to represent a month. A deviation of about 2.0°F (1.1°C) is predicted after one month of load aggregation. This error diminishes quickly after a few hours and goes practically to zero after about 48 hours of hourly history.

To reduce or eliminate the error at the start of the hourly load period, the aggregation algorithm is modified so that a user-defined minimum hourly history period during which only short time-step load superposition occurs always precedes the computation of the current average borehole temperature. To expand on the above example of determining the temperature at the 2281st hour, the incorporation of a minimum hourly history period of say 96 hours would result in that the third load aggregation would not be performed since the remaining number of hours (91 hours) is less than

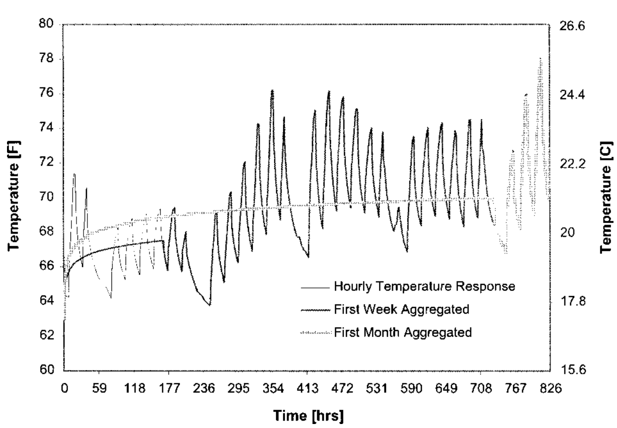

Figure 5 *Comparison of temperature responses based on hourly loads with temperature responses obtained through load aggregation for first week average and first month average loads for a typical case.*

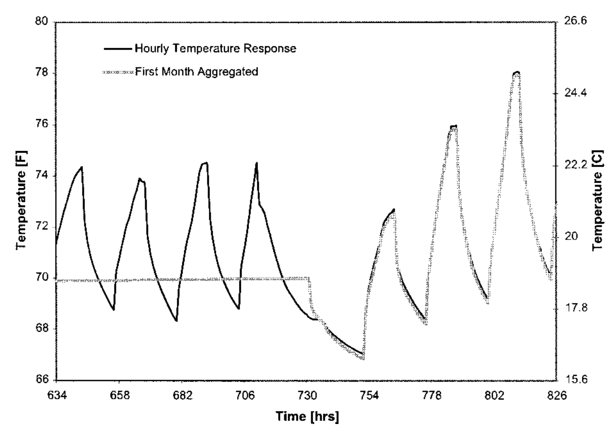

Figure 6 *Hourly history versus aggregated history for a typical case.*

the required minimum hourly history period specified. In this case, note that $q_{1460} = \overline{q}_2$.

$$T_{2281} = T_{ground} + \sum_{m=1}^{2} \left[\frac{(\overline{q}_m - \overline{q}_{m-1})}{2\pi k_{ground}} g\left(\frac{t_{2281} - t_{(730m-730)}}{t_s}, \frac{r_b}{H} \right) \right] + \sum_{n=1461}^{2281} \left[\frac{(q_n - q_{n-1})}{2\pi k_{ground}} g\left(\frac{t_{2281} - t_{n-1}}{t_s}, \frac{r_b}{H} \right) \right]. \quad (9)$$

Figure 7 shows a comparison of minimum hourly history periods of 24, 192, and 730 hours. In the short time-step model, the minimum hourly history period is an adjustable variable that can easily be changed by the user. Currently, the algorithm uses a minimum hourly history period of 192 hours.

As greater hourly history periods are selected the differences in average borehole temperature predictions between the load-aggregated and non–load-aggregated schemes decrease. If a minimum hourly history period of 8760 hours were to be selected for an annual simulation the block load-aggregated scheme collapses into the hourly scheme without any change in the total simulation time. However, with a minimum hourly history period of 192 hours, the computation time is reduced to approximately 10% of the time required for the nonaggregated scheme for an annual simulation. For a 20-year simulation, the computation time of the aggregated scheme is reduced to significantly less than 1% of the nonaggregated scheme due to the factorial relationship between the number of superposition calculations and the number of time steps.

Summary of Short Time Step Response Factor Model

The load aggregation algorithm has been developed above by example. The algorithm may be summarized for an hourly simulation in pseudocode as follows:

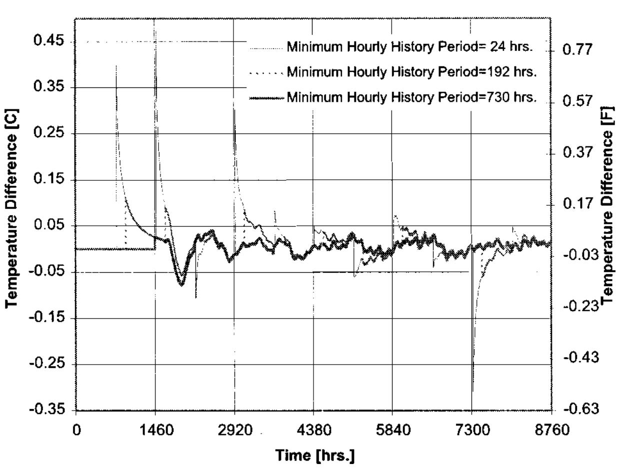

Figure 7 *Comparison of the error in predicted exiting temperatures for various minimum hourly history periods.*

Define An Aggregated Load Block (p) in Hours and Minimum Hourly History Period.

Read Borehole Geometry Parameters (Number of Boreholes, Borehole Depth, Radius.)

Read Ground and Fluid Thermal Properties (Ground Conductivity, Volumetric Specific Heat of Ground and Heat Transfer Fluid).

Read Short and Long Time-Step g-functions (Interpolate and store all g-functions for the simulation).

Do Loop n=1 to Number of Hours (nf)

Compute the Ground Load for the Current Time Step using Entering Fluid Temperature (For the first time step $T_{FarField}$ may be used).

Compute the Number of the Current Aggregated Load Block (calb) using the Minimum Hourly History Period and Aggregate Ground Loads in Blocks up to the Current Load Block.

If (The Current Time is Less Than the Sum of a Single Aggregated Load Block and Minimum Hourly History Period) → No Load Aggregation.

Compute Average Borehole Temperature by Superposition of the decomposed Short time Step Load Profile using the corresponding g-functions with Equation 10.

$$T_{nf} = T_{ground} + \sum_{n=1}^{nf} \left[\frac{(q_n - q_{n-1})}{2\pi k_{ground}} g\left(\frac{t_{nf} - t_{n-1}}{t_s}, \frac{r_b}{H} \right) \right]. \quad (10)$$

Else → Load Aggregation.

If (The Difference between the Current Time and the product of the Number of the Current Aggregated Load Block and a Single Aggregated Load Block is Greater Than Minimum Hourly History Period) →

Compute Long Time-Step Temperature Differences by Superposition of Aggregated Loads using the corresponding g-functions.

Compute Short Time-Step Temperature Differences by Superposition of Hourly Loads using the Short Time-Step g-functions.

Compute Average Borehole Temperature by Superposition of the Short and Long Time Step Temperature Differences with Equation 11.

$$T_{nf} = T_{ground} + \sum_{m=1}^{calb} \left[\frac{(\overline{q}_m - \overline{q}_{m-1})}{2\pi k_{ground}} g\left(\frac{t_{nf} - t_{(pm-p)}}{t_s}, \frac{r_b}{H} \right) \right]$$

$$+ \sum_{n=nf-[(calb)p]}^{nf} \left[\frac{(q_n - q_{n-1})}{2\pi k_{ground}} g\left(\frac{t_{nf} - t_{n-1}}{t_s}, \frac{r_b}{H} \right) \right]. \quad (11)$$

```
    Else →
Use Equation 12 to compute the Average
Borehole Temperature by Superposition of
the Short and Long Time Step Temperature.
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$$T_{nf} = T_{ground} + \sum_{m=1}^{calb-1} \left[\frac{(\overline{q}_m - \overline{q}_{m-1})}{2\pi k_{ground}} g\left(\frac{t_{nf} - t_{(pm-p)}}{t_s}, \frac{r_b}{H} \right) \right]$$

$$+ \sum_{n=nf-[(calb-1)p]}^{nf} \left[\frac{(q_n - q_{n-1})}{2\pi k_{ground}} g\left(\frac{t_{nf} - t_{n-1}}{t_s}, \frac{r_b}{H} \right) \right]. \quad (12)$$

```
    Endif
Endif
Continue Loop
```

Ground Loop Heat Exchanger Component Model for TRNSYS

TRNSYS (Klein et al. 1996) is a transient system simulation program with a modular structure that allows the use of externally developed mathematical simulation models for system components. Utilizing the short time-step g functions, a TRNSYS component model of the ground loop heat exchanger was developed. The component model allows for an annual or longer hourly building simulation incorporating a ground-coupled heat pump system. Shorter time steps than an hour may be used.

Although TRNSYS solves all equations in the component models simultaneously, TRNSYS models are cast in input/output form. The ground loop heat exchanger component model was formulated with the entering fluid temperature and the fluid mass flow rate as input variables. However, the short time step model described above assumed the heat rejection/extraction per unit length of borehole was the fundamental input variable and yields the average fluid temperature. Therefore, it is necessary for the component model to internally solve for the average fluid temperature, exit fluid temperature and heat rejection/extraction per unit length of borehole simultaneously.

The model parameters include the borehole depth and radius, the ground and fluid thermal properties, far-field ground temperature, the borehole thermal resistance, and the complete set of response factors. The short time-step model output variables for the current time step are the fluid temperature exiting the ground loop, the mass flow rate of the fluid, and the average fluid temperature of the borehole field. A schematic of model inputs and outputs are given in Figure 8 along with model internal parameters.

Because the model uses historical data, all hourly ground loads are stored. Some computational speed improvement is achieved by precomputing all hourly g functions up to 8760 hours and passing them to the load aggregation and superposition routine as required for all time steps up to the current time step.

Simple Heat Pump Component Model for TRNSYS

A simple water-to-air heat pump component model, which also reads hourly building loads from a file, was developed for testing purposes. Although TRNSYS has detailed heat pump and building models, a simplified model was desired. The simplified model is intended to be used with the total hourly building loads and the heat pump model merely translates an hourly building load into an exiting fluid temperature based on the entering fluid temperature and mass flow rate.

The model uses a quadratic curve fit of manufacturer's catalog data to compute the heat of rejection in cooling mode, the heat of extraction in heating mode, and the heat pump power consumption as functions of entering water temperature. Outputs provided by the model include the exiting fluid temperature, the power consumption, and the fluid mass flow rate.

EXAMPLE APPLICATION

An example application is provided using an actual building located in downtown Tulsa, Oklahoma that represents a cooling dominated commercial building. It is a four-story, 45,000-ft^2 (4182 m^2) office building, with a peak load of approximately 100 tons (352 kW). The building is to be gutted and completely renovated. The renovation will include an atrium with double pane, low-emissivity glass. However, the building loads are dominated by internal heat gains and the solar heat gains from the atrium skylights. Consequently, the building requires cooling year round. The building load profile is shown in Figure 9. (Heat rejection is shown as positive, heat extraction as negative.)

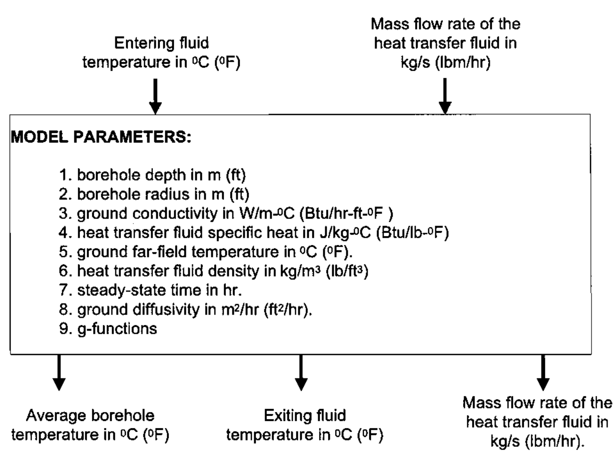

Figure 8 *TRNSYS short time-step component model configuration.*

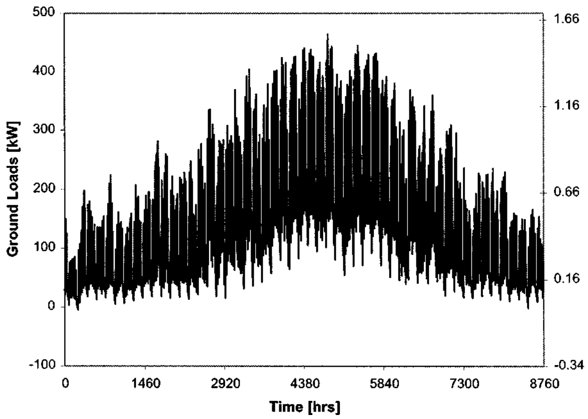

Figure 9 *Annual hourly building load profile for the example building in Tulsa.*

The modeled borehole loop field consists of 100 boreholes, each 250 ft (76.2 m) deep, arranged in a 10×10 rectangular configuration and spaced 25 ft (7.6 m). A very simple schematic of the system configuration is given in Figure 10.

The average borehole temperature increases, as expected from the building loads, during the summer months as a result of higher cooling needs for those months. The average temperature response to the loads in Figure 9 is shown in Figure 11. Entering and exiting water temperatures for the months of January and July are provided in Figures 12 and 13.

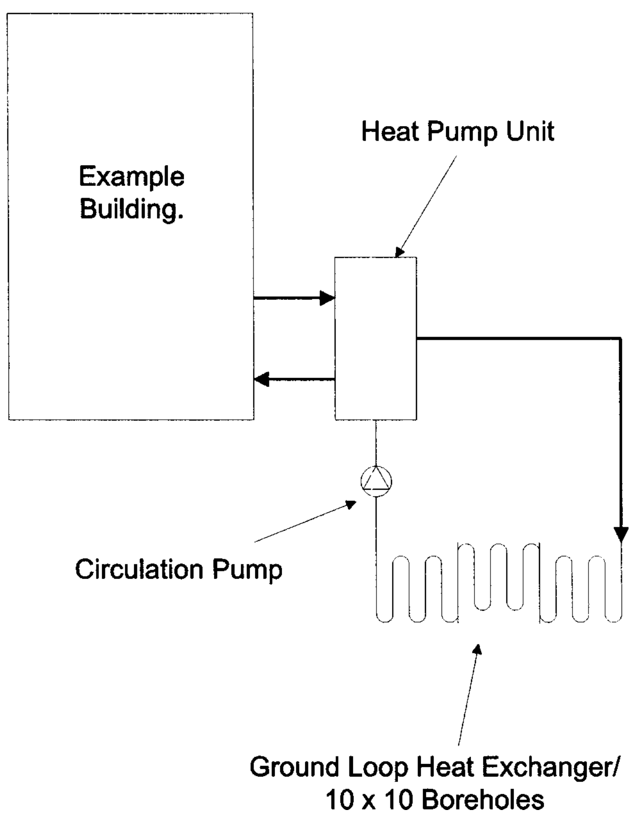

Figure 10 *A very simple schematic configuration of the example building setup.*

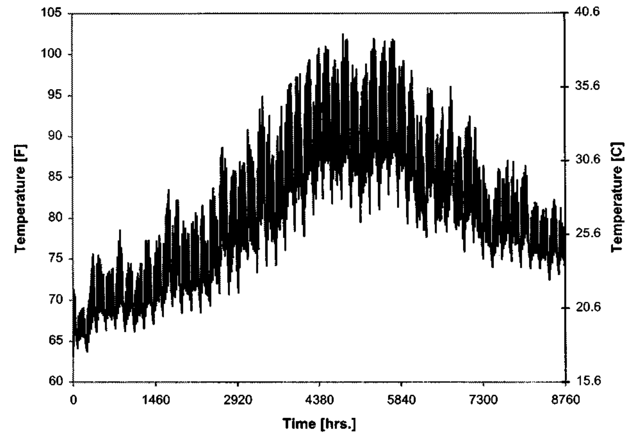

Figure 11 *Hourly average borehole temperature profile for the example building in Tulsa, Oklahoma as a result of an annual simulation with TRNSYS.*

A comparison of the average borehole temperatures between the first and the 8760th hour of simulation show that the average borehole temperature after the first year is about 10°F (5.6°C) higher than at the start of the simulation. In later years, the fluid temperatures would rise to the point that the heat pumps would fail. This is indicative of ground loop heat exchanger being undersized. Either the ground loop heat exchanger needs to be larger, or supplemental heat rejection units such as cooling towers could be used to avoid long-term thermal buildup in the ground. While existing design tools can be used to estimate the correct size of the ground loop heat exchanger, a short time-step model of the ground loop heat exchanger, coupled with component models of the building, heat pumps, and cooling tower can predict the impact of cooling tower sizing and operating strategy on the ground loop heat exchanger size and system operating cost.

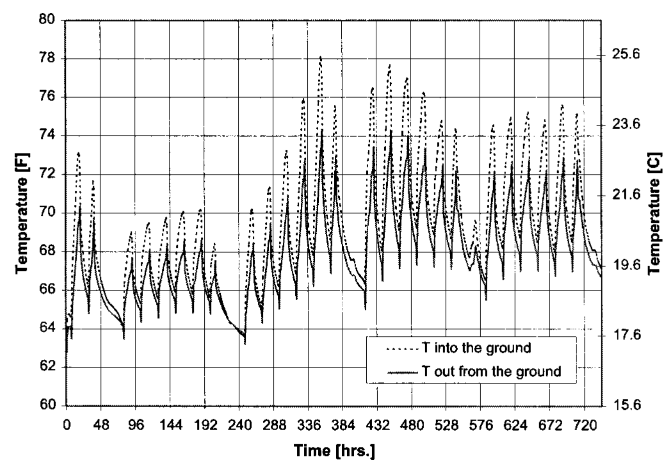

Figure 12 *Hourly input and output temperatures to the ground during the month of January for the example building in Tulsa, Oklahoma.*

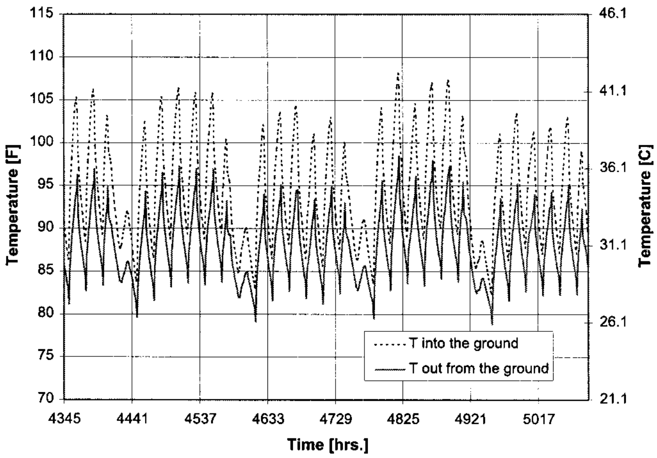

Figure 13 *Hourly input and output temperatures to the ground during the month of July for the example building in Tulsa, Oklahoma.*

CONCLUSIONS AND RECOMMENDATIONS

Short time-step nondimensional temperature response factors have been developed based on a transient two-dimensional finite volume numerical model. The short time-step response factors are a very useful extension of the long time-step response factors developed by Eskilson allowing for an hour-by-hour or shorter time-step evaluation of system energy consumption and electrical demand. A more accurate and detailed assessment of the short-term behavior of ground-coupled heat pump systems can thus be made for design of ground loop heat exchangers, energy analysis of ground source heat pump systems, and design of hybrid ground source systems.

The short time-step response factors were used in conjunction with a load aggregation algorithm to develop a short time-step ground loop heat exchanger model, which was cast as a component model for TRNSYS. A simple, but useful, model of a water-to-air heat pump was developed. An annual hourly simulation is performed for an example building to demonstrate the models.

Finally, it would be highly desirable to have an experimental validation for the model. Unfortunately, to date, little, if any, suitable[2] data exist and the collection of such data is not trivial. However, parts of the short time-step temperature response factors model have been verified analytically (Yavuzturk et al. 1999) and experimentally (Hellstrom 1991). Nevertheless, comprehensive experimental validation would be useful.

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NOMENCLATURE

α = diffusivity (ft^2/hr [m^2/hr])
D = diameter (ft [m])
g = g function
H = Borehole depth (ft [m])
k = conductivity (Btu/hr-ft-F [W/m-C])
Pr = Prandtl number
Q = Heat transfer rate (Btu/hr-ft [W/m])
q = Heat transfer rate for short time-steps (Btu/hr-ft [W/m])
r = radius (ft [m])
R = thermal resistance (F per Btu/hr-ft [C per W/m])
Re = Reynolds number
T = temperature (F [C])
t = time (hr)

Array Variables and Subscripts

b = borehole
i = index of any time steps
in = inside
l = index of hourly time steps
m = index for the load aggregated time blocks
out = outside
s = steady-state

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[2]In the authors' opinion, a suitable data set would include a high-quality independent measurement of the ground thermal properties at the site. Monitoring of the system, which would include accurate measurements of the loop flow rate and inlet and outlet temperatures, would have to commence at the beginning of the system operation.

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