Limitations of Using Uniform Heat Flux Assumptions in Sizing Vertical Borehole Heat Exchanger Fields

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Abstract

Models of ground heat exchangers and the surrounding ground are essential for design, optimization and energy analysis of ground heat exchangers (GHE) used with ground source heat pump (GSHP) systems. These models can account for ground thermal response of GHE in timescales ranging from hours to as long as years. A commonly used, computationally efficient method utilizes a dimensionless response function known as a g-function. The heat extraction and rejection loads on the GHE are devolved into a series of step inputs, then the g-function is used to determine the response due to each step input, and the temperature responses are superimposed to determine the evolution of borehole temperatures with time. G-functions for specific borehole configurations are calculated by superposition of single borehole responses and have been obtained with both numerical and analytical approaches. The numerical approach offers the most flexibility and does not require the approximation that all boreholes have uniform heat fluxes. However, the numerical approach requires significant computation time and the only practical approach at present is to pre-compute a library of g-functions. This paper examines the limitations of the uniform heat flux assumption as embodied in one analytical approach with a parametric study.

Keywords - Ground Source Heat Pumps; Borehole Heat Exchangers; Finite Line Source Theory; g-functions

1. Introduction

Ground source heat pump (GSHP) systems are a widely-used energy-efficient technology for meeting the heating and/or cooling demands of residential and commercial buildings. Because of the long time constant of the ground, design tools that account for heat extraction and/or rejection over a number of years are required for sizing purposes. A computer simulation of the heat flow in the ground is used to predict the temperature rise and fall over time of the fluid temperature leaving the ground heat exchanger. One approach [1] involves the use of a dimensionless response function known as a g-function. The heat extraction and rejection loads on the GHE are devolved into a series of step inputs, then the g-function is used to determine the response due to each step input, and the temperature responses are superimposed to determine the evolution of borehole temperatures with time.

G-functions for specific borehole configurations are calculated by superposition of single borehole responses and have been obtained with both numerical and analytical approaches. The numerical approach offers the most flexibility and does not require the approximation that all boreholes have uniform heat fluxes. However, the numerical approach requires significant computation time and the only practical approach at present is to pre-compute a library of g-functions. A number of analytical approaches [2-5] have been published in recent years. This paper examines the limitations of the uniform heat flux assumption as embodied in one such approach [5] with a parametric study.

The analytical approach assumes that all boreholes have uniform heat flux. Therefore, as the heat flux in the different boreholes becomes more non-uniform, we expect that the analytical approach will become less accurate. In general, non-uniform heat fluxes are caused by edge effects - e.g. in densely packed rectangular grids, the outer boreholes will have higher heat fluxes, as time goes on, than the inner boreholes. Therefore, we might expect that the larger the number of boreholes in each direction, the higher the non-uniformity and the higher the error in the simplified analytical approach. Likewise, end effects create non-uniformity along the boreholes and we expect these to be more important as the borehole depth decreases.

Claesson and Javed [5] verified their approach by comparing the response predicted with their analytical g-functions to those predicted with Eskilson's g-functions for configurations with 1, 3, and 9 boreholes. Small errors can be observed after 10,000 hours; the errors clearly increase with increasing number of boreholes and with increasing time. Fossa, et al. [4] also noted a discrepancy between their analytical solution and Eskilson's results; they surmised that it could be caused by the different assumptions regarding uniform heat flux.

This paper further investigates the differences between the two approaches in order to characterize the error caused by using the simplified analytical approach. Ultimately, we hope to determine generally applicable guidelines that could indicate when the use of the simplified analytical approach is acceptable. This paper focuses mainly on rectangular arrays of boreholes, but also looks briefly at other shapes such as open rectangles and double L ("L2") configurations.

2. Methodology

The methodology used for this comparison involves application of the analytical approach and the numerical approach to generate g-functions; the g-functions developed with both approaches are then used in a bore field sizing procedure. A parametric study then examines the differences for a range of configurations. The numerical and analytical approaches, the bore field sizing procedure and the parametric study are described below.

The numerical approach of Eskilson [1] for computing the long term response is based on a two dimensional finite difference model of heat extraction boreholes in radial-axial coordinates. Spatial superposition, which accounts for thermal interference between neighboring boreholes, was used for the analysis of multiple boreholes. The temperature response is then converted to non-dimensional form to be stored as a g-function for a specific borehole configuration. Because this process is time consuming, a library of g-functions is developed for different borehole configurations. For each configuration, g-functions are stored for different ratios of borehole spacing (B) to depth (H); to use these at other combinations, they are logarithmically interpolated by the sizing software. [7]

The analytical approach described by Claesson and Javed [5] is implemented in a computer program. The non-dimensional form of the response is divided into two regions, a short numerical response, which is computed numerically as described by Javed and Claesson [6]. A suitable breaking time is chosen to connect the short term response to the long term response solution. Any time between 10 hours and 1000 hours can be chosen for the breaking time as explained in [5]; 50 hours was used here.

The sizing procedure [7] simulates the temperature response of the ground heat exchanger using either the numerical or the analytical approach. It adjusts the required borehole length iteratively to meet the required maximum and minimum entering fluid temperature limits. The sizing requires the monthly total and monthly peak loads as inputs from the user. Three different building/location combinations presented in [8] were initially considered for this study – a school building in Memphis, Tennessee and Burlington, Vermont, and an office building in Miami, Florida. After beginning the study, we found that the Miami office building, having the highest annual imbalance, also always had the highest errors in the sizing calculations. So, we only used the Miami office building for the parametric study in order to estimate the maximum likely error. The monthly loads are obtained from the building energy analysis program EnergyPlus. The peak load analysis tool developed by Cullin and Spitler [9] was used for determining the duration of the peak loads. Because a range of building loads are needed for this investigation, monthly loads and monthly peak loads determined from a sample building are scaled up and down with a multiplier, representing a larger or smaller office building.

The heat pump entering fluid temperature limits for sizing the borehole heat exchanger fields were chosen to be 44°C maximum and 19°C minimum. Miami has an undisturbed ground temperature of 25°C and these limits are consistent with the recommendations provided in [10].

Because the building loads were scaled, the total system flow rate was also scaled automatically to 0.045 liters/second of water per kilowatt of peak cooling. Typical parameters were chosen for the ground properties and ground heat exchanger:

- Ground thermal conductivity: 1.73 W/m·K
- Ground volumetric heat capacity: 2160 KJ/K-m^3
- A single U-tube with standard bentonite grout in a 110 mm diameter borehole, with borehole resistance of 0.208 K/(W/m)

3. Results

The section presents comparisons of the numerical and analytical g-functions for different borehole configurations. Then, uncertainties in the sizing results are discussed. The parametric study then looks at the effects of borehole heat exchanger field size, borehole spacing and borehole depth on the error. Finally, an error caused by interpolation between database entries with the numerical method is discussed.

As discussed in the introduction, the analytical approach is expected to over predict the ground thermal response for more densely packed borehole fields. Figure 1 shows g-functions for square arrays of boreholes with a ratio of borehole spacing (B) to depth (H) of B/H=0.0625. The g-functions are the temperature responses of the borehole heat exchanger field to a step heat input, multiplied by the ratio of $2\pi k$ to the step heat input magnitude, where k is the thermal conductivity. The time is non-dimensionalized by dividing by the steady-state time scale, t_s [1] which is $H^2/9$ divided by the thermal diffusivity. As expected, as the number of boreholes in each dimension increases, and hence the uniformity decreases, the difference between the analytical g-function and the numerical g-function increases.

Before presenting the sizing comparisons, a brief discussion of the uncertainties in the sizing calculations is necessary. We have identified three types of uncertainties that can affect these comparisons.

1. The convergence criteria on fluid temperature limits to which bore field is sized. By using sensitivity analysis, the uncertainty in the sized depth due to the convergence criteria (± 0.01°C) was quite small; - ±0.3% or less.
2. Uncertainty due to the interpolation of g-functions in the numerical approach. As mentioned above, the numerical approach relies on a database of g-functions for specific configurations and discrete values of B/H which are interpolated logarithmically. In the results shown below in Figures 2-6, only these discrete values of B/H have been used to eliminate any uncertainty due to this interpolation.
3. Resolution of the time domain in the numerical integration of finite line source solution. The number of intervals used for integrating the weighting function was also found to affect the magnitude of the computed analytical g-functions. For a 10x10 array of boreholes with 5 m spacing and 100 m depth, 400 intervals were sufficient. However, for cases with much deeper boreholes, more intervals

were needed. In the present study, 1000 intervals were used to precisely compute the g-function, and the resulting uncertainty is negligible. In practice, it may be desirable to use fewer intervals in order to speed computation time.

The uncertainty in the oversizing shown in Figures 2-6 is then ±0.3%.

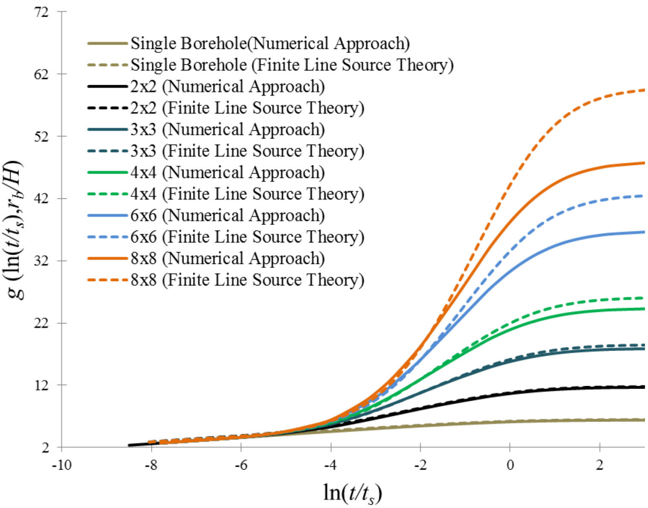

Figure 1 Long time step *g-function* comparisons (*B/H*=0.0625)

Figure 1 showed the deviation between numerically-calculated and analytically-determined g-functions for a range of square borehole arrays increases as the number of boreholes in close proximity increases and as time goes on. Figure 2 shows the resulting increase in sizing error with an increase in both the number of boreholes and the number of years considered for sizing. An acceptable limit on oversizing of a simplified method may depend on the situation, but if we take 5% as an acceptable limit, we can see that the simplified analytical approach would be acceptable, for a 20 or 30 year sizing period, for arrays up to 5x5 in size, at 5 m spacing and 100 m depth.

The oversizing error was also found to vary with borehole depth. In general, the deeper the borehole field, the less the error in the finite line source-derived g-functions. Figure 3 shows these errors for a 10x10 rectangular borehole array and two other configurations that would have same outer dimensions, but that have fewer boreholes, and, hence, less borehole-to-borehole interference. The 10x10 rectangular configuration shows the highest sensitivity to the borehole depth. It is expected that the end effects, which are treated approximately by the finite line source method, are more important when the boreholes are shorter. Our interpretation of these results is that the end effects are also more important in more densely packed borehole fields.

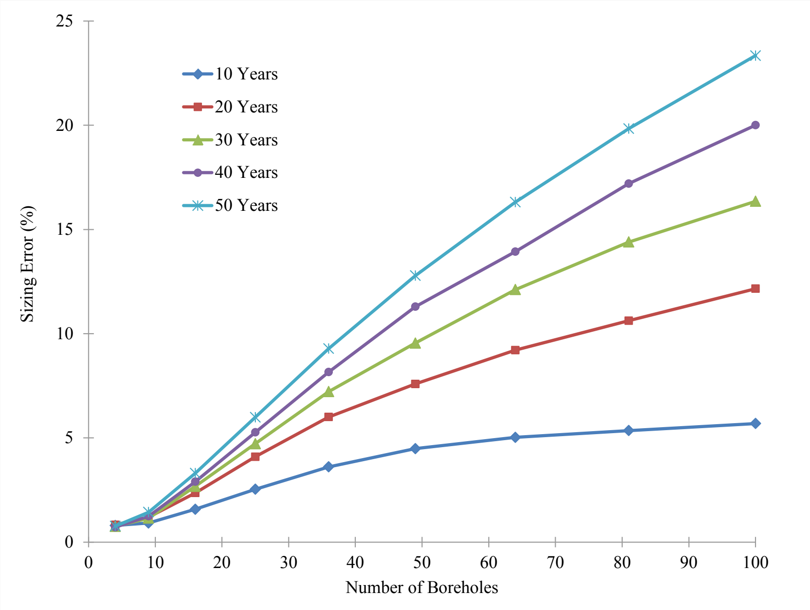

Figure 2 Sizing error for square borehole fields (5m spacing, 100 m deep boreholes)

Figure 4 shows the impact of spacing on the oversizing error. Decreasing the spacing increases the error, and increasing the number of boreholes in close proximity exacerbates this effect.

Beyond showing the effects of borehole spacing, depth, and number of boreholes, we sought a method to quantify these effects and looked at derived parameters that might help better characterize the errors. Three parameters were investigated:

- The mean spacing computed by obtaining the average distance of separation between the individual boreholes.
- The aspect ratio – i.e. the ratio of the number of boreholes in one direction to the number of boreholes in the other direction.
- The ratio of the number of interior boreholes (InteriorBH) to total number of boreholes (TotalBH).

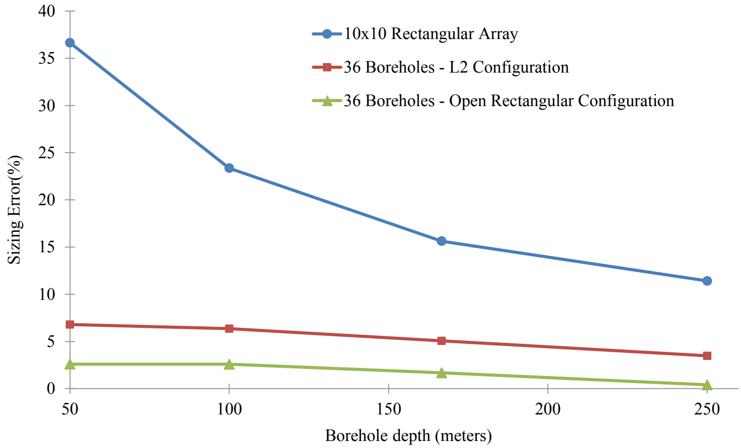

Figure 3 Sensitivity of sizing error to borehole depth – (50 years, 5m spacing)

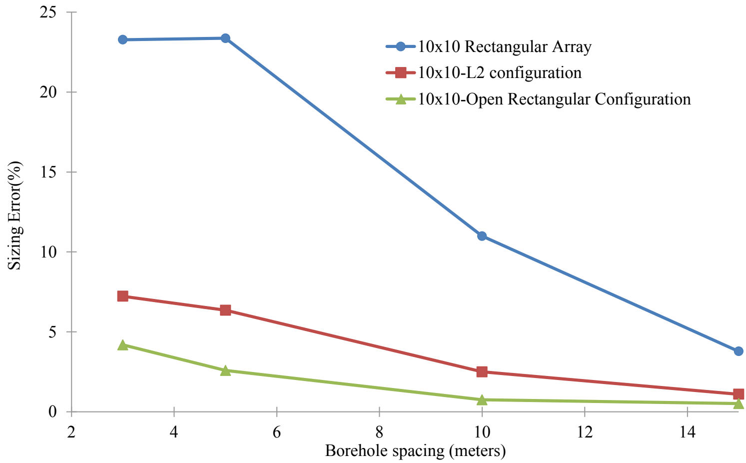

Figure 4 Error for three configurations (80 m deep boreholes, 50 years)

The oversizing error correlated most closely to the third parameter, the ratio of the interior boreholes to total boreholes. Figure 5 shows oversizing errors for a 20-year sizing period for a range of different rectangular configurations; they are grouped into different ranges of borehole numbers. E.g., the 35-36 range includes 2x18, 3x12, 4x9, and 5x7 configurations. This comparison is made for a 20-year sizing period. A horizontal bar has been drawn at the 5% oversizing level. While there is some variance between the different groups of boreholes, we might say that, roughly, if a maximum 5% oversizing is acceptable, then the analytical g-functions would be acceptable for fields of up to 64 boreholes with ratios of interior to total boreholes of less than 0.4. A similar analysis for a 30-year sizing period (Figure 6) suggests that the analytical g-functions would be acceptable for fields of up to 64 boreholes with ratios of 0.3 or less if the acceptable oversizing limit were taken as 6%. It should be kept in mind that these errors are for a worst-case annual ground heat exchange imbalance of a cooling-only office building in Miami. Errors for more balanced cases would be much smaller.

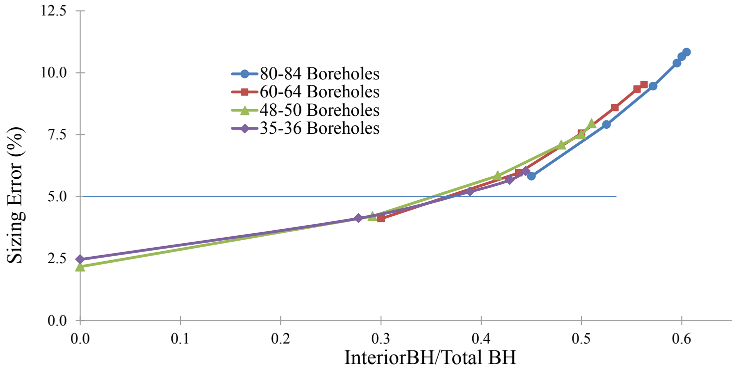

Figure 5 Sizing error versus interior to total borehole ratio (20 yrs., 5 m spacing, 100 m depth)

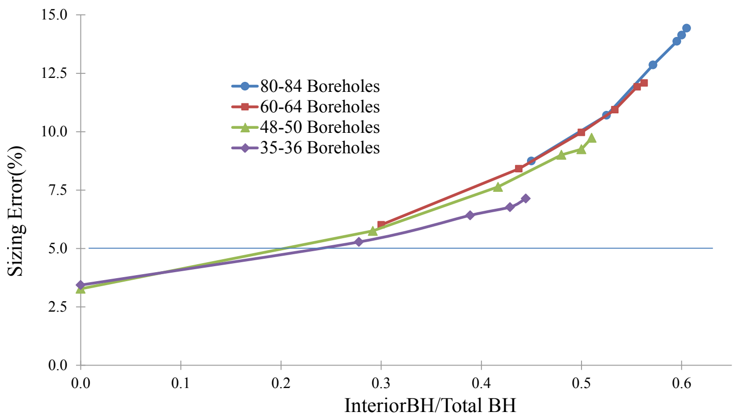

Figure 6 Sizing error versus interior to total borehole ratio (30 yrs., 5 m spacing, 100 m depth)

Finally, one of the unexpected findings of this study is related to the logarithmic interpolation between different *B/H* ratios of the database of numerical g-functions. When we investigated this interpolation, we first made an estimate of the error by comparing the g-function values at various times by plotting the values vs. the *B/H* ratio on a log scale; comparing a smooth curve to the straight-line logarithmic interpolation suggests that the logarithmic interpolation may overestimate values of g-functions by as much as 4%. While any peak temperature prediction will depend on quite a few values of g-function, a 4% overprediction of g-function suggests that the resulting size of the ground heat exchanger could be oversized by as much as 4%. Then, when compared to the analytical g-function, an overestimate of numerically-determined size could lead to a 4% underprediction in the sizing error. This is illustrated in Figure 7, which shows the oversizing for a 10x10 array. Here, +4%/-0.3% error bars are shown on the interpolated points. It appears that the 4% estimate of possible error is probably too high, but this is a subject for further investigation.

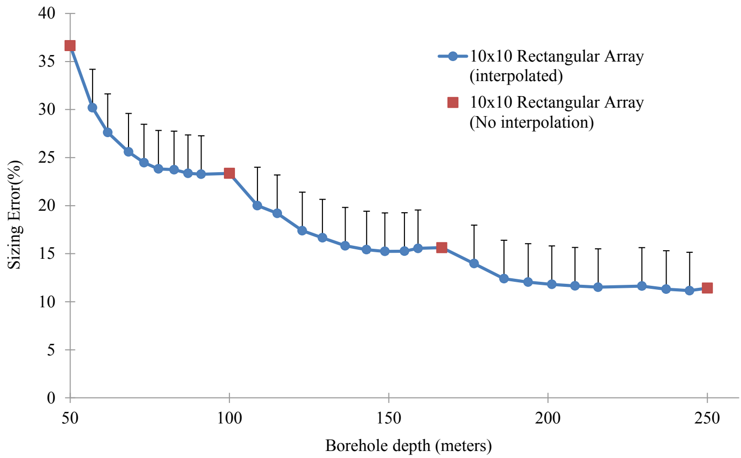

Figure 7 Sizing error due to interpolation

4. Conclusions

This paper has investigated maximum errors in sizing of ground heat exchangers due to use of a simpler analytical approximation (the finite line source method) for computing g-functions. While the actual errors will depend on the annual imbalance of heating and cooling loads, the maximum errors for a worst case load profile depend on borehole depth, spacing, and overall density. For borehole fields of up to 64 boreholes, with a borehole depth of around 100 m and borehole spacing of 5 m, the maximum oversizing error will be around 5% when the ratio of interior to total boreholes is less than 0.4 for a 20-year sizing period. For a 30-year sizing period, the error will be around 6% or less if the ratio is 0.3 or less. However, for many real life cases, the error will be substantially less and the analytical g-functions could be used for a wider range of configurations. A recommendation for future research is that this error be quantified for a larger number of cases and an equation-fit be developed that can estimate the error as a function of borehole spacing, depth, configuration, and the building load profile. This will allow better judgment as to when the finite line source method is appropriate.

An unexpected finding of this work is that interpolating between database entries of g-function values leads to an error in sizing, perhaps as high as a few percent. This should be corrected by improving the interpolation scheme.

Finally, it should be noted that this work is narrowly focused on the differences between methodologies used to analyze pure conduction heat transfer with uniform soil thermal properties. There are many other real-world effects that have some (usually minor) effect on the performance. These include groundwater movement, moisture transport in unsaturated soils, and depth varying ground thermal properties.

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