Modeling of Vertical Ground Loop Heat Exchangers with Variable Convective Resistance and Thermal Mass of the Fluid

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1. BACKGROUND

The ability to predict the short-term behavior of ground loop heat exchangers (GLHE) is critical to the design and energy analysis of ground source heat pump (GSHP) systems. Thermal load profiles vary significantly from building to building – GLHE designs can be dominated by long-term heat build-up or short-term peak loads. In some extreme cases, where the GLHE design is dominated by short-term peak loads, temperatures in the GLHE can rise rapidly; say 5-10ºC in one to two hours. For such short-term peak loads, the thermal mass of the fluid can significantly dampen the temperature response of the ground loop. The over prediction of the temperature rise (or fall) in turn can cause an over prediction of the required GLHE length. Furthermore, the temperature response can be damped by the fluid in the rest of the system, in addition to the fluid in the borehole. The temperature response also has a secondary impact on the predicted energy consumption of the system, as the COP of the heat pump varies with entering fluid temperature. Therefore, it is desirable to be able to model the short-term behavior accurately.

In GSHP systems, antifreeze mixtures are often used as a heat transfer fluid. Generally, the flow rate in the GLHE is designed so as to ensure turbulent flow in the tube to guarantee a low convective heat transfer resistance. However, for some antifreeze types, the large increase in viscosity as the temperature decreases may result in transition to laminar flow, or require an otherwise unnecessarily high system flow rate. For example, at 20°C, the viscosity of 20% weight concentration propylene glycol is 0.0022 Pa.s and the density is 1021 kg/m^3. At –5°C, the viscosity increases to 0.0057 Pa.s and the density increases 1026 kg/m^3. That means, with the same volumetric flow rate, the Reynolds number at –5°C is only about 39% of the value at 20°C. If this results in transition from turbulent to laminar flow, the convective resistance will increase significantly. In order to evaluate the trade-offs between high system flow rates and occasional excursions into the laminar regime, it is desirable to include the effects of varying convective resistance in the GLHE model.

Analytical solutions, e.g. line source model (Ingersoll and Plass 1948), cylindrical heat source (Carslaw and Jaeger 1947; Hellstrom 1991; Kavanaugh 1995; Bernier 2001), and other analytical solutions(Hellstrom 1991; Sutton, et al. 2002), have been used for dimensioning vertical ground heat exchangers. Direct numerical solutions, e.g. finite difference, finite volume, finite element (Muraya 1994; Rottmayer et al. 1997; Zeng, et al. 2003; Al-Khoury, et al. 2005) have been used to model ground heat exchangers, but, due to computational time requirements, are not useful for incorporation into a building simulation program with hourly or shorter time steps.

A third approach, which allows for computationally efficient simulation, involves the development of response functions, often called g-functions, which allow the ground heat exchangers to be modeled with a time series. Eskilson (1987) developed g-functions for long time step, and later a short time step response factor method was developed by Yavuzturk and Spitler (1999). While this model can simulate the heat transfer of ground loop heat exchanger in any time scale, it could not model varying thermal resistance and fluid thermal mass.

The objective of the paper is to describe the development of a new short time-step model for vertical GLHE. Like the Yavuzturk and Spitler (1999) model, it is an extension to the original long time-step Eskilson model (Eskilson 1987). However, whereas that model used a short time-step g-function to account for short time-step effects, the model described in the paper replaces the response function approach at short time-steps with a one-dimensional numerical model, which explicitly accounts for the thermal mass of the fluid and the convective resistance as a function of flow rate, fluid mixture, and fluid temperature. The paper also describes an approach for representing the two-dimensional borehole geometry in one dimension, utilizing the multipole method (Bennet, et al. 1987) for calculating the thermal resistance of the grout, while maintaining the correct thermal mass of the grout. Validation and implementation of the model, along with an example application are also described.

2. METHODOLOGY

The new ground loop heat exchanger model is based partly on the long time g-functions developed by Eskilson (1987) and partly on one-dimensional numerical model used to determine the short time response. As the method developed by Eskilson is the basis for the model of ground loop heat exchanger, it will be described first. Eskilson's approach to the problem of determining the temperature distribution around a borehole is based on a hybrid model combining analytical and numerical solution techniques. A two-dimensional numerical calculation is made using transient finite-difference equations on a radial-axial coordinate system for a single borehole in homogeneous ground with constant initial and boundary conditions. The thermal capacitance of the individual borehole elements such as the tube wall and the grout are neglected. The temperature fields from a single borehole are superimposed in space to obtain the response from the whole borehole field.

The temperature response of the borehole field is converted to a set of non-dimensional temperature response factors, called g-functions. The g-function allows the calculation of the temperature change at the borehole wall in response to a step heat input for a time step. Once the response of the borehole field to a single step heat pulse is represented with a g-function, the response to any arbitrary heat rejection/extraction function can be determined by devolving the heat rejection/extraction into a series of step functions, and superimposing the response to each step function. This process is graphically demonstrated in Figure 1 for four months of heat rejection.

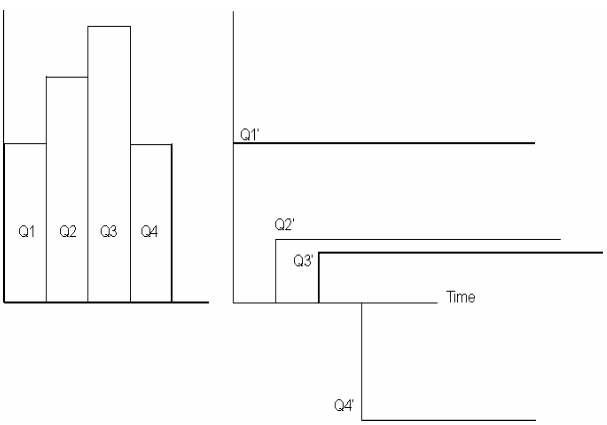

Figure 1 Superposition of Piece-Wise Linear Step Heat Inputs in Time

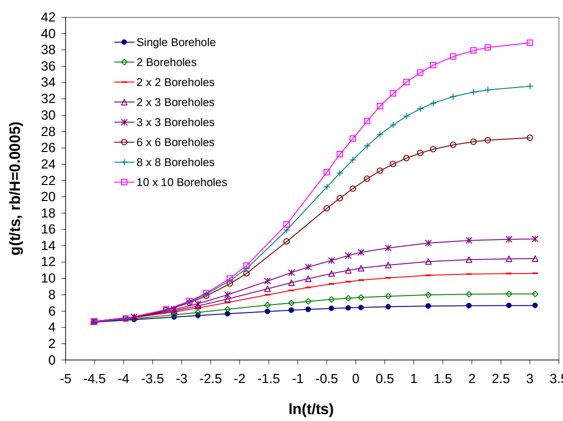

Figure 2 Temperature response factors (g-functions) for various multiple borehole configurations compared to the temperature response curve for a single borehole.

The basic heat pulse from zero to Q1 is applied for the entire duration of the four months and is effective as Q1'=Q1. The subsequent pulses are superimposed as Q2'=Q2-Q1 effective for 3 months, Q3'=Q3-Q2 effective for 2 months and finally Q4'=Q4-Q3 effective for 1 month. Thus, the borehole wall temperature at any time can be determined by adding the responses of the four step functions. Mathematically, the superposition gives the borehole wall temperature at the end of the nth time period as:

$$T_{borehole} = T_{ground} + \sum_{i=1}^{n} \frac{(Q_i - Q_{i-1})}{2\pi k} g\left(\frac{t_n - t_{n-1}}{t_s}, \frac{r_b}{H}\right) \quad (1)$$

Where t is the time, s; t_s is the time scale = $H^2/9\alpha$; H is the borehole depth, m; k is the ground thermal conductivity, W/m-K; $T_{borehole}$ is the average borehole temperature in $^{\circ}$C; T_{ground} is the undisturbed ground temperature in $^{\circ}$C; Q is the step heat rejection pulse, W/m; r_b is the borehole radius in m; and i is the index to denote the end of a time step (the end of the 1^{st} hour or 2^{nd} month etc.)

Figure 2 shows the temperature response factor curves (g-functions) plotted versus non-dimensional time for various multiple borehole configurations and a single borehole. The g-functions in Figure 2 correspond to borehole configurations with a fixed ratio of 0.1 between the borehole spacing and the borehole depth. The thermal interaction between the boreholes is stronger as the number of boreholes in the field is increased and as the time of operation increases.

The detailed numerical model used in developing the long time-step g-functions approximates the borehole as a line source of finite length, so that the borehole end effects can be considered. The approximation of the borehole as a finite-length line source has the resultant problem that it is only valid for times estimated by Eskilson to be greater than 5 $r^2_{Borehole}/\alpha$. For a typical borehole, that might imply times from 3 to 6 hours. However, much of the data developed by Eskilson does not cover periods of less than a month. (For a heavy, saturated soil and a 76 m deep borehole, the g-function for the single borehole presented in Figure 2 is only applicable for times in excess of 60 days.)

One-Dimensional Numerical Method for Short Time-Step Response

Yavuzturk and Spitler (Yavuzturk and Spitler 1999) extended Eskilson's long time-step model to short time steps by developing short time-step g-functions with a two-dimensional (radial-angular) finite volume method, which utilized an automated gridding procedure and a "pie-sector" representation of the U-tubes. Because the short time-step g-function represented the response of the entire ground heat exchanger, it necessarily utilized a fixed convective resistance. The authors later found it necessary (Yavuzturk and Spitler 2001) to modify the model to include variable convective resistance, but this was done at the expense of modeling the thermal mass of the fluid in the borehole.

In order to simultaneously account for variable convective resistance and thermal mass in the borehole, a one-dimensional numerical model is used directly to compute the short time-step response. This is integrated with Eskilson's long time-step model. By careful control of the one-dimensional model parameters, the model is able to give acceptably accurate short-term response, without the computational time that would be required to run such a model continuously throughout the simulation. The representation of a single U-tube ground heat exchanger with a one-dimensional model is illustrated in Figures 3 and 4. At short times, end effects can be neglected.

The one-dimensional model has a fluid core, an equivalent convective resistance layer, a tube layer, a grout layer, and is surrounded by the ground. In order to get near-identical results to the more detailed two-dimensional model, it is important to specify the one-dimensional geometry and thermal properties in an "equivalent" manner:

- Equivalent volumetric thermal mass of fluid -- the cross-sectional area of the fluid multiplied by the density and specific heat in the 1-d model should be equivalent to the actual 2-d geometry. It is also possible to account for additional fluid in the system outside of the borehole by increasing the thermal mass to account for the total volume of fluid in the system.
- The 1-d outer tube diameter is equal to $\sqrt{2}$ the outer diameter of the actual U-tube. This maintains the grout thermal mass and is also a standard approximation used for cylinder-source models of U-tubes.
- A thin artificial layer in the 1-d model between the fluid and grout represents the convective resistance. An equivalent conductivity is determined which gives the same resistance as the actual convection coefficient at the inside of the U-tube.
- The resistance between the inner tube wall and the borehole wall is determined with the multipole method. (Bennet et al. 1987) A single equivalent conductivity for the tube layer and grout layer is based on this resistance. However, the tube grout layers have different thermal masses, set to match their actual individual thermal masses. The layer representing the tube wall in the 1-d model has the same thickness as the actual tube wall. The grout layer thickness is chosen to maintain the borehole diameter.

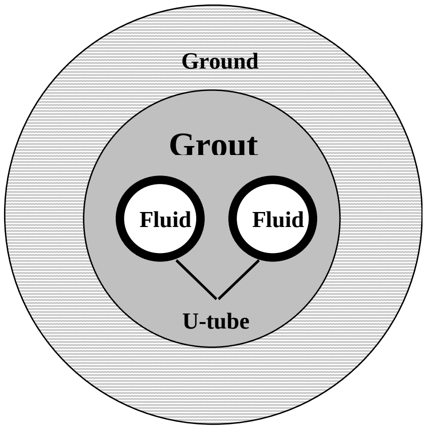

Figure 3 A schematic drawing of a borehole system

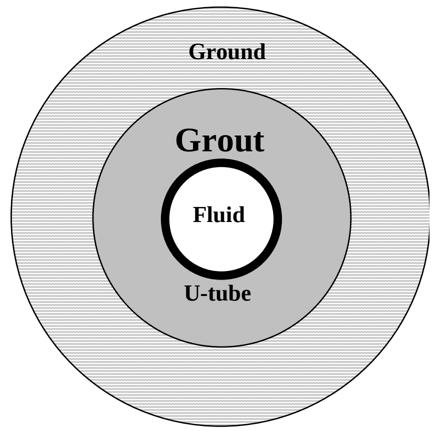

Figure 4 A schematic drawing of the simplified one-dimension model

The gridding procedure and specification of the parameters are discussed in more detail below.

Geometry and gridding procedure

The geometry of the one-dimensional model is shown in Figure 5. Working from the outside in:

- r_{far} the far-field radius is set to 10 m; the soil region is represented by 500 cells
- $r_{borehole}$ is set to the actual borehole radius; the grout region is represented by 27 cells
- $r_{out,tube}$ is set to $\sqrt{2}$ times the outer radius of the actual U-tube; the tube region is represented by 4 cells
- $r_{in,tube}$ is set to $r_{out,tube}$ - $\Delta r_{U\text{-}tube}$, where $\Delta r_{U\text{-}tube}$ is the actual U-tube wall thickness;
- $r_{in,convection}$ is set to $r_{in,tube}$ - ¼ $\Delta r_{U\text{-}tube}$; the artificial convection layer is represented by 1 cells
- r_{fluid} is set to $r_{in,convection}$ - ¾ $\Delta r_{U\text{-}tube}$; the artificial convection layer is represented by 3 cells. The fluid is represented as an annulus because it is assumed to have very high thermal conductivity, i.e. a lumped capacitance for which the internal distribution of temperature is uniform, and the actual heat input is then represented as a heat flux boundary condition.

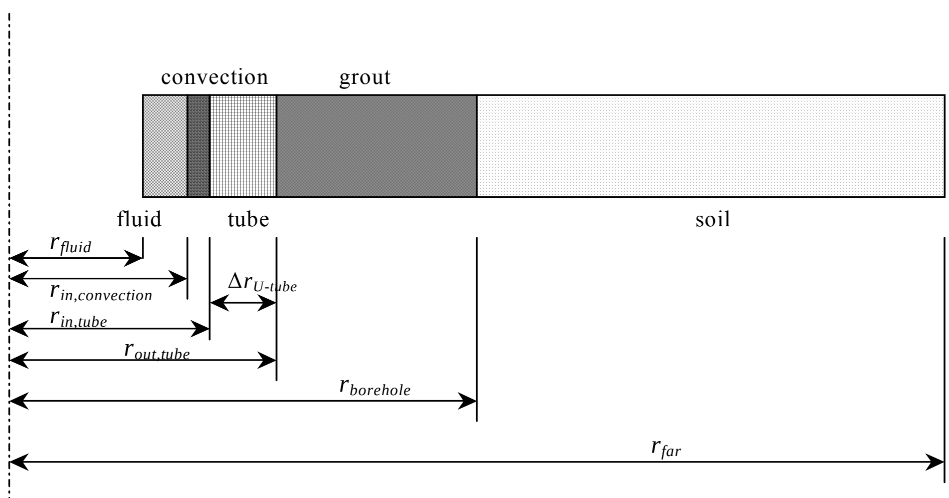

Figure 5 A schematic drawing of the simplified one-dimension model for the finite volume method

Equivalent volumetric thermal mass of fluid

In the ground loop heat exchanger system, fluid inside the U-tube and elsewhere in the system helps to damp the response to peak loads. Since this can be critical in the design of some systems, it is desirable to account for this in the model.

The equivalent thermal mass of the fluid can be calculated from:

$$\pi(r_{in,convection}^2 - r_{fluid}^2)C_{equiv,fluid} = 2\pi r_{actual,in,tube}^2 C_{fluid} \qquad (2)$$

Where:

$C_{equiv,fluid}$ = equivalent volumetric thermal capacity of fluid (J/m^{3}K)

C_{fluid} = volumetric thermal capacity of fluid (J/m^{3}K)

$r_{actual,in,tube}$ = inner radius of the actual U-tube (m)

The fluid at any cross-section in the U-tube is assumed to be at a uniform temperature. In the 1-d model, this is enforced by setting the thermal conductivity of the fluid to a high value (200 W/mK). Finally, the fluid inside the system, but outside the U-tube may be accounted for by multiplying the $C_{equiv,fluid}$ by the "Fluid Factor". A typical value for an actual system is two.

Equivalent thermal conductivity of tube and grout

In order to get near-identical results with the two-dimensional model, the borehole thermal resistance in the simplified one-dimensional model should be exactly equal to the thermal resistance of the actual two-dimensional borehole. The multipole method (Bennet et al. 1987) is used to calculate the thermal resistance between the borehole wall and the convective resistance, accounting for the differing grout and U-tube thermal conductivities. The multipole method is a highly accurate analytical method and has compared very well to a two-dimensional boundary-fitted coordinate finite volume numerical model. (Rees 2000)

The single equivalent conductivity assumed for the grout and tube layers can be calculated once the equivalent thermal resistance has been calculated with the multipole method:

$$k_{equiv,TG} = \frac{\ln(r_{borehole} / r_{in,tube})}{2\pi R_{equiv,TG}} \qquad (3)$$

Where:

$k_{equiv,TG}$ = equivalent conductivity of tube and grout layers (W/m-K)

$R_{equiv,TG}$ = equivalent thermal resistance (m-K/W) between the borehole wall and convective resistance

Equivalent conductivity for convection layer

The multipole method is also used to calculate the thermal resistance between the borehole wall and the fluid, (R_{BH}) as before, except including a specific value of the convective resistance. Then the equivalent thermal resistance of the artificial convection layer is:

$$R_{equiv,convection} = R_{BH} - R_{equiv,TG} \qquad (5)$$

The convective resistance is approximated as a conductive layer of thickness ¼ the U-tube wall thickness. The equivalent thermal conductivity of the convection layer ($k_{equiv,convection}$) can then be expressed as:

$$k_{equiv,convection} = \ln(r_{in,tube} / r_{in,convection}) / 2\pi R_{equiv,convection} \qquad (6)$$

The thermal mass of the convection heat transfer layer is set to a near-zero value. (1 J/m^{3}K)

Currently, the multipole method is called at each time step to calculate the borehole thermal resistance after the convection coefficient has been calculated. Gnielinski's (Hellstrom 1991) correlation is used to calculate the Nusselt Number when the flow in the U-tube is in turbulent, $\mathrm{Re} > 2300$; the Nusselt Number is set to 4.364 when the flow in the tube is in laminar, $\mathrm{Re} \le 2300$. Since this method will result in a discontinuity at the transition point, a linear correlation is used to smooth the Nusselt Number when $2200 \le \mathrm{Re} \le 2500$.

One-Dimensional Numerical Method Validation

Validation of the one-dimensional numerical method is highly desirable. Rees (2000) developed a boundary-fitted finite volume program (GEMS2D) that was used to validate the one-dimensional numerical method over a range of parameters. A single validation is illustrated here, with properties of a single borehole system operating under a step heat input as detailed in Table 1. For this case, the average fluid temperature in the borehole is plotted in Figure 6. The predictions of average fluid temperature compare well with an RMSE of 0.09ºC. A number of additional comparisons have given similar results.

Table1 Borehole Properties for Inter-model Validation

Borehole Diameter (m)	0.114	Soil Volumetric Heat Capacity (MJ/m^{3}K)	2.5
Borehole Length (m)	72	Grout Volumetric Heat Capacity (MJ/m^{3}K)	3.9
U-tube Inside Diameter (m)	0.02744	Tube Volumetric Heat Capacity (MJ/m^{3}K)	1.77
U-tube Outside Diameter (m)	0.03341	Fluid Convection Coefficient (MJ/m^{3}K)	1690
Shank Spacing (m)	0.01583	Step heat input (W/m)	40.4
Soil Conductivity (W/m-K)	2.5	Fluid Type	Water
Grout Conductivity (W/m-K)	0.7443	Average Fluid Temperature (°C)	20
U-Tube Cond. (W/m-K)	0.3895		

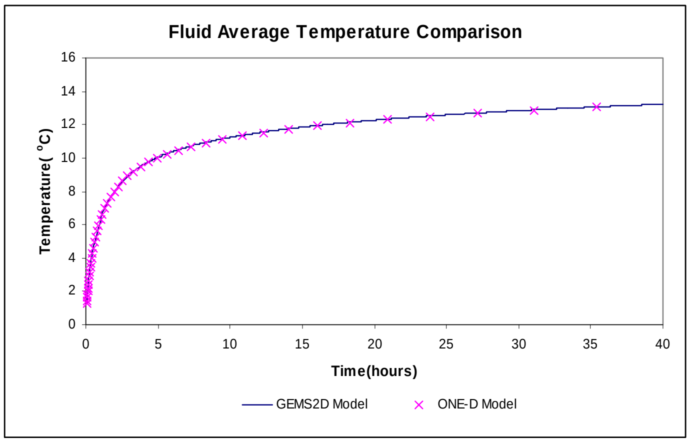

Figure 6 Average fluid temperature comparison

Using the one-dimensional numerical method, the fluid temperature response due to time-varying heat inputs can be obtained at any time step including short time steps (hourly or less). For cases where time-varying convective resistance is not of interest, this method can also be used to determine non-dimensional short time step g-functions.

3. MODEL IMPLEMENTATION

The new ground loop heat exchanger component model was completed in the HVACSIM+ package (Clark 1985). The model parameters include the number of boreholes, borehole depth and radius, U-tube configurations, the U-tube, the grout and the ground thermal properties, fluid type and fluid factor of the system, and the long time step g-functions. The model is formulated to take inlet temperature and mass flow rate as inputs, and give the outlet temperature as an output.

4. EXAMPLE APPLICATION

An example application is provided using an imaginary peak-load-dominant building in Birmingham, AL – a one-story, 547 m^2 church with a significant cooling or heating load of duration two hours peak occurring on a weekly basis. The building load was calculated using BLAST (BLAST 1986) and the load profile is shown in Figure 7 (heating load is positive and cooling load is negative). Eight Climate Master GSV/H070 water-to-air heat pumps with a nominal cooling capacity of 21 kW, capable of meeting the design capacity required for the church building. The heat pumps are modeled within HVACSIM+ using a simple component model that determines the heat pump COP as a polynomial function of entering fluid temperature and mass flow rate. The model then returns the exiting fluid temperature, which is an input to the ground heat exchanger model.

A 16-borehole ground heat exchanger, laid out in a 4x4 grid with boreholes 76.2 m deep, and other parameters shown in Table 1 is utilized. The total volumetric flow rate of the ground heat exchanger is 9.1 L/s. The undisturbed ground temperature in Birmingham is 18.3 $^{\circ}$C. In this system, the fluid factor of GSHP system is selected as 2 to account for the fluid in the distribution piping. The hourly entering and exiting fluid temperatures for the first year of operation are shown in Figure 8. The model is commonly used for multi-year simulations.

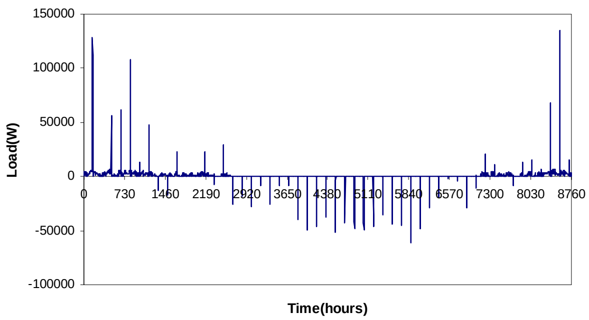

Figure 7 Annual hourly building load profiles for the church building in Birmingham, AL.

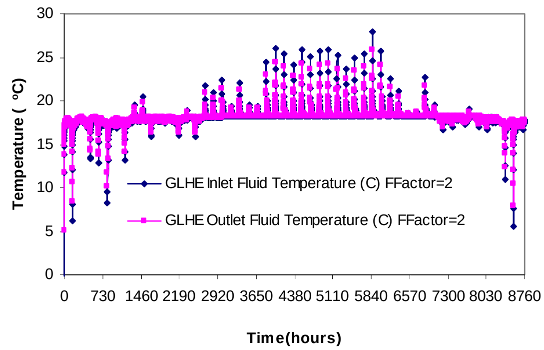

Figure 8 Hourly ground loop fluid temperature profiles for the church building in Birmingham.

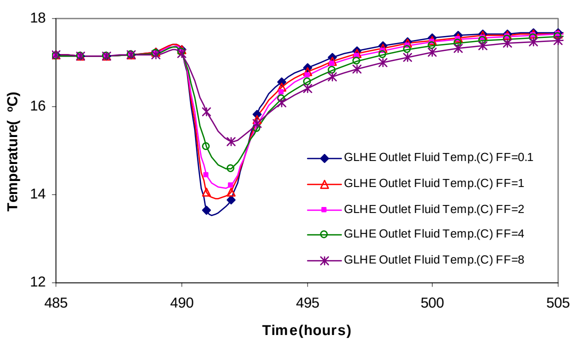

Figure 9 Detailed GLHE outlet fluid temperatures for different fluid factors

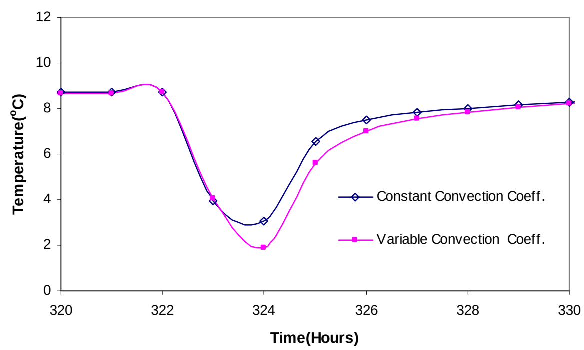

Figure 10 Detailed GLHE outlet fluid temperatures for different convection coefficient models

The effect of different fluid factor values can be seen in Figure 9, which shows 21 hours of operation that includes a peak heating period. The 0.1 value of fluid factor would be representative of a simulation that did not include the effect of the thermal fluid mass; the value of 1 would account only for the thermal mass of fluid in the borehole; the value of 2 would be typical if the thermal mass of fluid in the distribution portion of the system is included; larger values might be representative of a system with long pipe runs or where additional fluid storage is intentionally included. The differences shown here might be expected to give a relatively small difference in energy consumption prediction. However, as sizing of the ground heat exchangers can be highly sensitive to the peak temperature prediction, the difference in fluid factor can have a significant impact on the GLHE design. (Young 2004)

To illustrate the use of the model in a situation where variable convective resistance may be important, the church building is modeled in Detroit, with 36 boreholes in a 6x6 grid, and a 30% by weight mixture of propylene glycol and water. Figure 10 shows a case where the flow transitions from turbulent, Re=2519, to laminar, Re=1857. The constant convection coefficient curve uses the convection coefficient determined at 10ºC, 511 W/m^{2}K. The variable convection coefficient curve shows the temperatures when the convection coefficient drops to 68 W/ m^{2}K as a result of the flow falling into the laminar regime. Designers are often careful to keep the total flow rate of the system always in the turbulent regime; with this model, the actual impact of this practice can be weighed against alternatives.

4. CONCLUSION

A new short time-step model for vertical ground loop heat exchanger with variable convective resistance and thermal mass of the fluid was developed in this paper. The temperature response at short time-steps is calculated with a one-dimensional numerical model, which explicitly accounts for the thermal mass of the fluid and the convective resistance as a function of flow rate, fluid mixture, and fluid temperature. In this model, the multipole method is used to calibrate the one-dimensional resistances so that they always match the total two-dimensional resistance. At the same time, the thermal mass of the individual components is maintained in the one-dimensional model. By carefully controlling these parameters, the one-dimensional model compares very favorably to a detailed boundary-fitted coordinates finite volume model.

The new GLHE model was cast as a component model in HVACSIM+. An annual hourly simulation GSHP system in a church building was carried out to demonstrate this new GLHE model and illustrate the effects of thermal mass and variable convective resistance.

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