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A One-dimensional Approximation for Transient Multi-dimensional Conduction Heat Transfer in Building Envelopes

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SUMMARY:

While transient multi-dimensional analysis of conduction heat transfer in building envelopes is feasible using numerical methods, only one-dimensional analyses are commonly integrated into building simulation (design load calculation and energy analysis) programs. While a not insignificant amount of research has gone into methods for modeling transient multi-dimensional conduction heat transfer in building simulation programs, uptake into programs actually used by practitioners has been slow. There, are, perhaps, several reasons for this, beyond the purely technical reasons. These include the difficulty in creating a user interface that would allow specification of multi-dimensional constructions in an acceptable length of time and the relatively low significance of opaque envelope heat transfer to the overall cooling load or building energy consumption. Despite the current state-of-the-art in building simulation software, there is surprisingly little guidance for practitioners on how to approximate multi-dimensional walls in one-dimensional building simulation programs. This paper reviews previously-proposed methods for modeling transient multi-dimensional heat transfer in building envelopes, then looks at a relatively simple approximation and quantifies its performance for several typical two-dimensional walls against detailed numerical models.

1. Introduction

Building envelopes commonly include walls and roofs with multi-dimensional elements, e.g. studs, joists, and rafters. For steady-state analysis, methods for determining the U-factor are widely available, e.g. in the ASHRAE Handbook – Fundamentals Volume (ASHRAE 2001). For dynamic analyses, i.e. cooling load calculations and energy analysis, the effects of such elements on building envelope heat transfer have been the subjects of a number of research projects (references here). However, dynamic analysis procedures for multi-dimensional walls and roofs have not found their way into commonly utilized cooling load calculation methods or energy analysis procedures. For these, practitioners are left to make some sort of simplifying approximation, and somehow describe the wall or roof as a set of one-dimensional layers in order to use a one-dimensional analysis.

This is particularly important for walls with a large disparity in thermal conductivity of the wall elements. A prime example are walls with steel studs and the effect is commonly called a “thermal bridge.” For dynamic analyses, the effects are exacerbated by thermally massive elements. Thermal bridge effects result in localized multidimensional heat flow paths, which reduces the overall resistance and time lag benefits of the walls significantly.

Several recent works (Carpenter et al., 2003; Kosny and Christian, 1995; Kosny et al., 1997; Brown et al., 1998; Thomas and Antar, 1998; Kosny and Kossecka, 2002; Strachan et al., 1995) have indicated that one-dimensional models cannot accurately predict the heat transmission through building envelopes with thermally massive elements and large disparity in thermal conductivity of the wall elements. A study made by Kosny and Christian (1995) shows the thermal bridge effect of metal stud walls can reduce the thermal resistance of the center of cavity values by about 48%. A similar study made by Kosny and Kossecka (2002) shows that one-dimensional parallel path approximations of the overall resistances for concrete and steel frame walls with 20% frame factor (fraction of the area of the frames to the total area of the envelopes) and 80% insulation were about 27% and 44%, respectively, higher than that of a detailed analysis using the finite difference method. Numerical and experimental

investigations by Davies, et al. (1995) show that ignoring the edge effects can under predict the overall heat loss by over 10%.

In this paper, we first review recent research in the area, then go on to propose a simple approximation, and show its accuracy by comparison to detailed numerical models for several different wall configurations.

2. Multi-dimensional Heat Conduction Models

Commonly used building energy analysis and load calculation programs in the USA all use one-dimensional time series heat conduction models, either conduction transfer function (CTF) or response factor methods. Although they have not found their way into common use, a number of multi-dimensional heat conduction models have been proposed, and can be divided into two categories: numerical models and time series models.

Numerical methods (finite difference, finite volume, and finite element methods) have the advantage of modeling the heat conduction with reasonable accuracy while having the flexibility to include time-varying convection coefficients and thermo-physical properties. (Time series models can be used with time-varying convection coefficients, although they are often implemented with fixed convection coefficients.) The main drawback of the numerical methods is that the nodal equations need to be solved every time step. Moreover, the numerical methods require significant computer memory for storing outputs of no interest. Thus, numerical methods have not been the principal method of choice for modeling transient heat conduction in building energy analysis and load calculation programs. Notable exceptions include two UK-developed programs: ESP-r (Clarke 2001) and HTB2 (Lewis and Alexander 1990). An alternative approach, which might be considered similar to a coarse-grid numerical method, is the development of an equivalent RC network (Akander, et al. 1996 and Mao 1998) that is tuned to have an equivalent dynamic response to a detailed numerical model. The following discussion reviews different categories of time series models of multi-dimensional heat conduction intended for use with building simulation programs.

2.1. State Space Method

The state space method (Seem, 1987; Strand, 1995) utilizes a spatial finite difference discretization of the building wall. Instead of discretizing the problem in time and solving the finite difference equations numerically for a certain set of time-varying boundary conditions, a state space approach is used to develop relationships between the states at different times. An analytical solution of the resulting set of equations produces conduction transfer function coefficients. Strand (1995) extended Seem's methodology to include embedded heat sources or sinks, e.g. embedded hydronic tubing. The main challenge in implementing a multidimensional state space model is that, for general use, it requires a general user interface and gridding methodology. This is a non-trivial task, and likely the reason why the methodology has not seen wider use.

2.2. Equivalent wall method

The equivalent wall method generates one-dimensional conduction transfer function coefficients for a fictitious homogeneous layered wall that replicates the dynamic response of real composite wall (Kosny and Kossecka, 2002; Kossecka, 1998; and Carpenter et al., 2003). In the equivalent wall method, an iterative technique is used in order to match the dynamic response of the fictitious wall with that of the actual multi-dimensional wall. The dynamic response of the actual multi-dimensional wall is determined with a numerical method (e.g. finite difference) computer program (Kossecka and Kosny, 1998; Carpenter et al., 2003). An obvious advantage of the equivalent wall method is that it can generate transfer function coefficients in one-dimensional form for any complex real composite wall. Alternatively, it can generate a fictitious wall description that can be used in any computer program that accepts a layer-by-layer description of a wall. In spite of this, the equivalent wall method has two shortcomings. First, the multi-dimensional response factors are generated using a separate standalone finite difference heat conduction program, which has the capability to generate complex grids. Thus, incorporating the equivalent wall method into an existing building energy analysis and load calculation program requires development of an independent multi-dimensional numerical method transient heat conduction sub-program and a user interface that will actually be used by practicing engineers. Second, the iterative technique is only described as a manual trial-and-error type method. No algorithm has yet been given that can be automated.

2.3. CTF Coefficient Generation with Finite Difference Model

Burch et al., (1992) demonstrated a finite difference analysis based model of determining CTF coefficients. This model generates CTF coefficients in one-dimensional form from a dynamic response analysis of the multi-dimensional building envelope. The method involves predicting the heat transfer response of a wall for a triangular pulse excitation of the temperature boundary conditions. Then, the CTF coefficients are derived from the response factors. The authors also presented a mathematical expression for removing the convection film coefficient from the CTF coefficients for direct use in the existing building energy analysis and load calculation programs. However, CTF coefficients are sensitive to the magnitude of the convection coefficient; hence the authors recommend further investigation of the sensitivity. Like the equivalent wall method, this method requires a separate numerical method program or sub-program to generate CTF coefficients.

2.4. Summary

Three types of multi-dimensional heat conduction models have been presented. The first type, state space methods, discretizes the wall, but solves the equations analytically to develop one-dimensional conduction transfer function coefficients. The second method, the equivalent wall method, develops a fictitious, one-dimensional wall description, prior to development of one-dimensional CTF coefficients. The third method uses a finite difference model to develop response factors and then converts the response factors to CTF coefficients.

For several reasons, these methods have not yet been implemented in commonly utilized load calculation and energy analysis procedures. While this is desirable, it may take some time before any of these methodologies are both available and feasible for use by practicing engineers. In the meantime, it is useful to provide some guidance as to what practicing engineers should do now. The rest of this paper proposes a method that is feasible for use by practicing engineers.

3. Equivalent Homogeneous Layer Method

The proposed method for representing multi-dimensional walls and roofs in one-dimensional layer-by-layer descriptions involves the substitution of a homogenous layer with the same thickness for the multi-dimensional layer. The homogenous layer is chosen such that it gives the correct overall R value for the wall using the best method available. We have taken the recommendations from the ASHRAE Handbook – Fundamentals Volume as the best method available for each case; e.g. for a wood stud wall, the parallel path method is used; for a metal stud wall, the modified zone method is used. Then, the density and specific heat are chosen to maintain the correct thermal mass of the layer. As a step-by-step procedure, it may be stated as follows:

- 1) Obtain the steady state overall R value of the wall by using the method recommended for the particular type of the wall. These methods are described in Chapter 25 of the ASHRAE Handbook – Fundamentals Volume (ASHRAE 2001).
- 2) Obtain the effective R-value of the homogeneous layer by subtracting the R values of the single dimensional layers from the overall R value of the wall determined in (1).
- 3) The equivalent conductivity of the homogeneous layer is obtained by dividing the thickness of the homogeneous layer with the resistance of the homogeneous layer.
- 4) The density of the homogeneous layer is the sum of the densities of the materials constituting the multidimensional layer multiplied by their volume fractions.
- 5) Similarly, the specific heat of the homogeneous layer is the sum of the product of the specific heat, density and the volume fraction of the materials in the multidimensional layer.

Once the thickness, conductivity, density and specific heat of the equivalent homogeneous layer have been determined, the wall may be described in a load calculation or energy analysis tool. In most cases, either conduction transfer function coefficients or response factors will be calculated and used in the analysis.

4. Validation of the Homogeneous Layer Method

Preliminary tests of the homogeneous layer method were made by comparing conductive heat fluxes computed with conduction transfer functions, generated with the homogeneous layer approximation, to

conduction heat fluxes calculated with a full two-dimensional representation of the construction in ANSYS. The conduction heat fluxes were driven by a sinusoidal exterior air temperature, coupled to the wall surface by a convective resistance. The methodology, constructions, and results are briefly described below; a fuller account may be found in Karambakkam (2004).

4.1. Walls

Three construction types were analyzed – a wood stud wall, a metal stud wall, and a concrete block wall. The wood stud wall consists of a layer of siding, a layer of plywood, a layer with fibreglass insulation and wood studs in parallel, and a layer of gypsum board, as shown in Figure 1, with the properties given in Table 1. The composite layer has 38 mm wide wood studs, 600 mm on center. For the wood stud wall, the ASHRAE Handbook -- Fundamentals Volume (2001) recommends the parallel path method for the calculation of the U-value. When convective resistances at the outside surface ($0.04 \text{ m}^2\text{-K/W}$) and inside surface ($0.18 \text{ m}^2\text{-K/W}$) are added, this gives an air-to-air U-value of $0.441 \text{ W/m}^2\text{-K}$ resulting in a homogeneous wall construction as described in Table 2.

TABLE 1: Construction of Wood Stud Wall.

| Layer Name [Numbers in brackets correspond to labels in Figure 1] | Specific Heat (kJ/kg-K) | Conductivity (W/m-K) | Thickness (mm) | Density (Kg/m ³) |
|---|-------------------------|----------------------|----------------|------------------------------|
| ½" Wood siding [1] | 1.255 | 0.072 | 12.7 | 544.0 |
| ½" Plywood (Path 1) [2] | 1.213 | 0.115 | 12.7 | 544.0 |
| 3.5" R-11 fiberglass (Path 2) [3] | 0.962 | 0.046 | 88.9 | 84.8 |
| 2x4 Wood studs [4] | 1.632 | 0.114 | 88.9 | 576.0 |
| ½" Gypsum [5] | 1.088 | 0.16 | 12.7 | 800.0 |

TABLE 2: One-dimensional Representation of Wood Stud Wall with Homogeneous Layer Approx.

| Layer Name | Specific Heat (kJ/kg-K) | Conductivity (W/m-K) | Thickness (mm) | Density (Kg/m ³) |
|-------------------|-------------------------|----------------------|----------------|------------------------------|
| ½" Wood siding | 1.255 | 0.072 | 12.7 | 544.0 |
| ½" Plywood | 1.213 | 0.115 | 12.7 | 544.0 |
| Homogeneous layer | 1.171 | 0.051 | 88.9 | 115.5 |
| ½" Gypsum | 1.088 | 0.160 | 12.7 | 800.0 |

The metal stud wall is similar in configuration to the wood stud wall, except that the wood studs are replaced with 16 gauge metal studs, which are 1.5 mm thick, and "C" shaped, with 38 mm wide flanges. The U-factor, including the inside and outside convective resistances and calculated with the modified zone method is $0.569 \text{ W/m}^2\text{-K}$. This gives the homogenous layer construction shown in Table 4.

The concrete block wall consisted solely of 200 mm thick concrete blocks with vermiculite filled cores. Table 5 gives the thermal properties of this wall and Figure 1 shows the geometry. Due to the considerable lateral heat transfer involved, the ASHRAE Handbook – Fundamentals Volume (2001) recommends the isothermal plane method for the calculation of U-value of a concrete block wall. This resulted in an air-to-air U-value of $1.006 \text{ W/m}^2\text{-K}$. Table 6 gives the properties of the resulting homogeneous wall construction. However, ANSYS gives a somewhat lower estimate of the U-value, $0.975 \text{ W/m}^2\text{-K}$. This is not unexpected – the isothermal plane method tends to be a conservative approximation, underpredicting the thermal resistance of the wall. If the ANSYS estimate for the U-factor is used instead of the isothermal plane method, the homogenous layer would have a conductivity of 0.182 W/m-K .

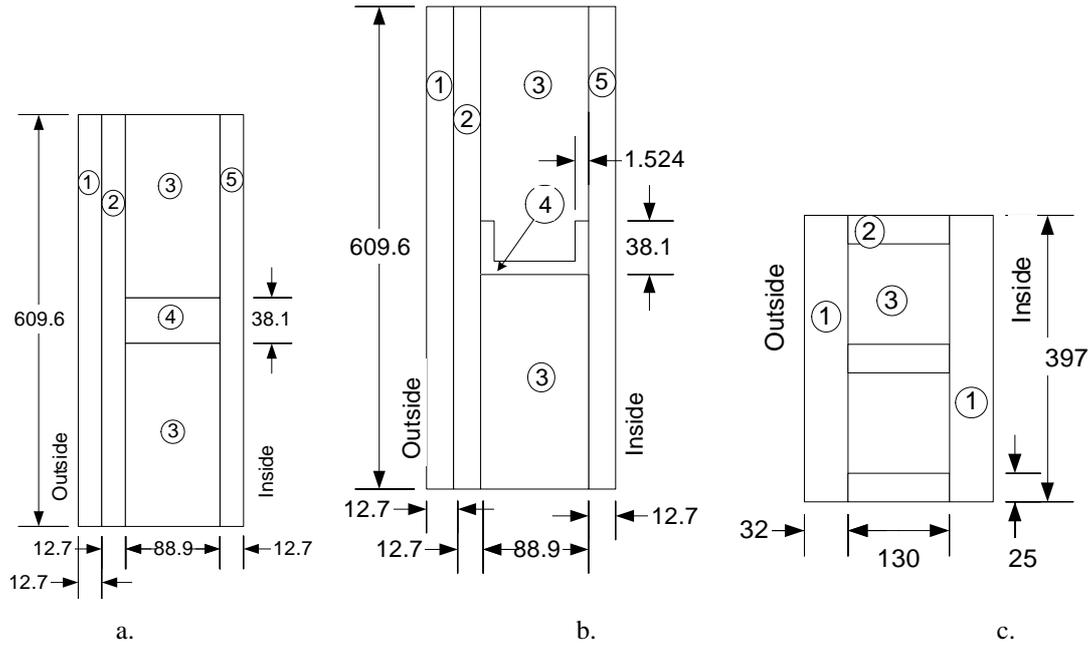


FIG. 1: Three wall types: a) wood stud; b) metal stud; c) concrete block. All dimensions in mm.

Table 3 Steel stud wall construction

| Layer Name [Numbers in brackets correspond to labels in Figure 1] | Specific Heat (kJ/kg-K) | Conductivity (W/m-K) | Thickness (mm) | Density (kg/m ³) |
|---|-------------------------|----------------------|----------------|------------------------------|
| ½" Wood siding [1] | 1.255 | 0.072 | 12.7 | 544 |
| ½" Plywood [2] | 1.213 | 0.115 | 12.7 | 544 |
| 3.5" R-11 fiberglass [3] | 0.962 | 0.046 | 88.9 | 84.8 |
| Steel studs [4] | 0.502 | 45.279 | 88.9 | 7840 |
| ½" Gypsum [5] | 1.088 | 0.16 | 12.7 | 800 |

Table 4 Homogeneous wall construction for steel stud wall with convective boundary condition.

| Layer Name | Specific Heat (kJ/kg-K) | Conductivity (W/m-K) | Thickness (mm) | Density (Kg/m ³) |
|-------------------|-------------------------|----------------------|----------------|------------------------------|
| ½" Wood siding | 1.255 | 0.072 | 12.7 | 544 |
| ½" Plywood | 1.213 | 0.115 | 12.7 | 544 |
| Homogeneous layer | 0.953 | 0.072 | 88.9 | 247.59 |
| ½" Gypsum | 1.088 | 0.16 | 12.7 | 800 |

Table 5 Concrete block wall Construction.

| Layer Name | Specific Heat (kJ/kg-K) | Conductivity (W/m-K) | Thickness (mm) | Density (Kg/m ³) |
|----------------------------|-------------------------|----------------------|----------------|------------------------------|
| Face shells [1] | 0.840 | 0.706 | 32 | 1380 |
| Web [2] | 0.840 | 0.706 | 130 | 1380 |
| Vermiculite insulation [3] | 1.340 | 0.068 | 130 | 110 |
| Face shells [1] | 0.840 | 0.706 | 32 | 1380 |

Table 6 Homogeneous wall construction concrete block.

| Layer Name | Specific Heat (kJ/kg-K) | Conductivity (W/m-K) | Thickness (mm) | Density (Kg/m ³) |
|-------------------|----------------------------|-------------------------|-------------------|---------------------------------|
| Face shells | 0.840 | 0.706 | 32 | 1380 |
| Homogeneous Layer | 0.967 | 0.189 | 130 | 350 |
| Face shells | 0.840 | 0.706 | 32 | 1380 |

4.2. Methodology

In ANSYS, a two-dimensional grid with triangular elements was used for each wall configuration. In each case, the gridding was iteratively refined to the point where the solution could be considered grid-independent. The actual number of elements depended on the configuration – the wood stud wall used 750 elements; the steel stud wall used 1147 elements; the concrete block wall used 1820 elements. A convective boundary condition was used at each wall surface. At the inside surface, the air temperature was 20°C. At the outside surface, a sinusoidal air temperature was defined with a mean value of 30°C, amplitude of 15°C, and period of 24 hours. The peak value of 45°C occurred at hour 6.

To make comparisons with the homogeneous layer approximation, the conduction transfer function coefficients were determined with a Laplace transform method implemented in a cooling and heating load calculation program. The CTF coefficients were applied in a spreadsheet with the same sinusoidal air temperature profile as used with ANSYS and conduction fluxes at the inside surface were determined.

4.3. Results

Conduction heat fluxes at the inside surface are calculated with ANSYS and CTFs derived with the equivalent homogeneous layer approximation. Results for the wood stud wall are plotted in Figure 2. The error in the peak flux of the homogeneous wall is about 0.6%. The flux response of the homogeneous layer approximation also lags that of ANSYS by one hour. Although the error is small, it may be thought of as having two sources – first, some error results from using the parallel path method to calculate the U-factor, and second, additional error is caused by the equivalent homogeneous layer approximation. The U-factor calculated with the parallel path method is 0.4414 W/m²-K. When the ANSYS simulation is repeated with steady boundary conditions, a U-factor of 0.4455 W/m²-K may be inferred from the results. Since the parallel path method U-factor is about 0.9% lower than the ANSYS U-factor, and the equivalent homogeneous layer method predicts a peak heat flux that is 0.6% higher, it may be inferred that the two errors counteract each other. Overall, the net effect of the two approximations appears to give a result that is adequate for design load calculations, which typically use a sinusoidal-type temperature profile. For energy calculations, which typically use real weather data, it seems likely that approximations are adequate, but further work would help confirm this.

Results for the metal stud wall are shown in Figure 2. For the metal stud wall it can be seen that the homogenous layer approximation underpredicts the peak heat flux by 0.7% as compared to ANSYS. Again, the thermal response time of the homogeneous wall lags that of ANSYS by 1 hour. The U-factor calculated by the modified zone method and used in the equivalent homogeneous layer approximation is 0.5688 W/m²-K. The U-factor inferred from ANSYS is 0.5645 W/m²-K. In this case, the U-factor used in the equivalent homogenous layer approximation is 0.8% higher than the ANSYS value, but the method predicts a peak flux that is 0.7% lower. Again, while the overall error is very small, it appears that the two sources of error counteract each other.

Results for the concrete block wall are shown in Figure 4. In this case, the homogeneous layer approximation overpredicts the peak conduction heat flux by 17.6%. Comparing the U-factors, the isothermal plane method gives a U-factor of 1.064 W/m²-K, while ANSYS gives a value of 0.975 W/m²-K. The isothermal plane method overpredicts the U-factor by 9.1% compared to ANSYS, and appears to make a substantial contribution to the overall error. Therefore, a second case was run with the equivalent homogeneous layer approximation. In this case, the ANSYS-predicted U-factor was used to determine the equivalent homogeneous layer approximations. The resulting conduction heat fluxes are labelled "Homogeneous Layer Approximation - ANSYS-adjusted U". Using this refined estimate of the U-factor, the homogeneous layer approximation only overpredicts the peak conduction heat flux by 8.0%. Interestingly, there is no discernible time lag in the conduction heat fluxes for this wall.

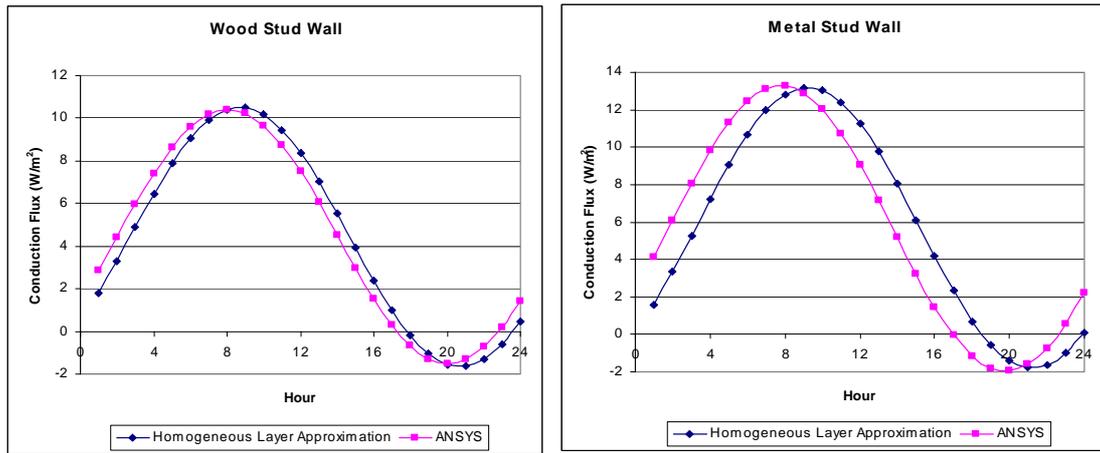


FIG. 2: Conduction heat fluxes for the two types of stud walls.

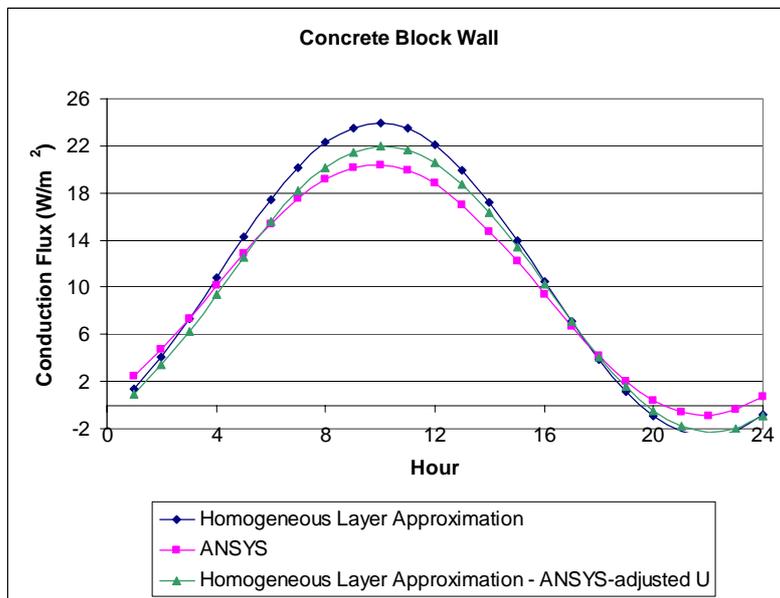


FIG. 3: Conduction heat fluxes for the concrete block wall.

5. Conclusions

Various proposed methodologies for modelling multi-dimensional conduction heat transfer in building simulation and load calculation programs have been briefly reviewed. All of the methods require, at the least, a gridding tool to spatially discretize the building wall or roof construction. Given the complexity of multi-dimensional building walls and roofs, this is a non-trivial task, and these methods have not been implemented in commonly-used building simulation and load calculation programs, though the implementation is highly desirable. Until such implementations are widely available, it is necessary for practicing engineers to have a method for approximating a multi-dimensional wall or roof in one-dimensional building simulation and load calculation programs.

Such a method, dubbed the “Equivalent Homogeneous Layer Method”, is proposed in this paper, and preliminary testing is presented. This method depends first on having a good method for determining the steady-state U-factor. Once the steady-state U-factor has been determined, the composite layer containing multiple elements is replaced with an equivalent homogeneous layer that maintains the layer thickness, overall U-factor of the wall, and thermal mass of the composite layer.

The method was tested against a detailed finite element program (ANSYS) for three different wall types, with a fixed interior air temperature and a sinusoidal exterior air temperature. For the two stud

walls, the error in peak heat flux was less than 1%. For the concrete block wall, the error in peak heat flux was 17.6%, although about half of the error can be attributed to the error in the U-factor computed with the isothermal planes method. Given the limited amount of testing, the proposed methodology appears to give adequate accuracy.

It is recognized that the limited amount of testing does not provide a complete validation of the methodology! Specifically, it is recommended that additional validation be done with more wall and roof types, different temperature profiles, and boundary conditions that include solar and thermal radiation. Furthermore, the method does not give any guidance for cases where there are multiple composite layers and it would be useful to develop a procedure to handle this situation.

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