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Optimum Duct Design for Variable Air Volume Systems, Part 2: Optimization of VAV Duct Systems

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ABSTRACT

Current duct design methods for variable air volume (VAV) systems are based on the use of peak constant airflow. However, VAV systems operate much of the time at an off-peak load condition and the impact of varying airflow rates to the sizing of duct systems has not been considered. This and a companion paper introduce an optimum duct design procedure for VAV systems to investigate the importance of the varying airflows to the system design. Hourly airflow requirements, part-load fan characteristics, and duct static pressure control are incorporated into the problem formulation. Constraints, such as discrete duct sizes and velocity limitations, are incorporated into the duct design procedure. In part 1, the domain of a VAV optimization problem is analyzed to define the problem characteristics and to suggest an optimization procedure. In part 2, the VAV duct design procedure is fully developed and applied to several VAV duct systems with different parameter values. The results are analyzed to compare duct design methods, and the effect of several factors that influence optimal design are investigated.

INTRODUCTION

The companion paper (Kim et al. 2002) defines optimum duct design for VAV systems, and the problem domain characteristics are examined in order to suggest a VAV optimization procedure. The objective function has a gradually sloping convex shape near the minimum, and the analysis shows that the problem appears to have only a global minimum. The Nelder and Mead downhill simplex method is successfully applied to the unconstrained VAV duct design problem to find optimum duct sizes. This study extends the problem domain analysis to find discrete optimum duct sizes in a constrained duct design problem. Design constraints for VAV duct systems are added as penalty terms to the objective function for any violation of the constraints. The Nelder and Mead downhill simplex method (Nelder and Mead 1965) is applied to search for a continuous design solution for the constrained duct design problem, and a penalty approach for integer programming is employed to impose penalties of discrete violation on the objective function to enforce the search to converge to nominal duct sizes. Duct fitting loss coefficients for different design conditions are sought using a duct fitting database program as described in ASHRAE (1993).

This VAV optimization procedure, the Nelder and Mead downhill simplex method with a penalty approach for integer programming, is applied to several VAV duct systems under different design conditions, such as different electric rates, different ductwork costs, and different system operating schemes. The optimized results are compared to those derived from equal friction, static regain, and the T-method. Throughout the comparison, the impact of varying airflow rates to the sizing of duct systems is investigated and the savings of the VAV optimization procedure are revealed.

VAV OPTIMIZATION PROCEDURE

The VAV optimization procedure is mainly composed of the preparation of airflow data, the evaluation of the objective function, and the generation of a design solution that includes continuous and discrete solutions. Figure 1 shows the overall optimum duct design procedure. First, time-varying airflow rate data for VAV duct systems can be provided by an hourly building simulation program. Second, the evaluation of the objective function requires fan selection, initial cost calculation, a search for fitting loss coefficients from the duct fitting

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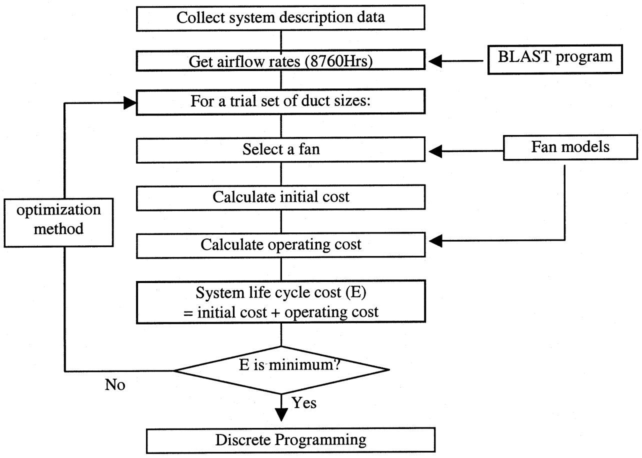


Figure 1 *Optimum duct design procedure.*

database, system pressure loss calculation, and operating cost calculation. They are explained in part 1 for unconstrained duct design problems. For constrained duct design problems, design constraints are defined and explained later in this section as to how they are incorporated in the optimization procedure. Third, an optimization method provides candidate design values for the estimation of system life-cycle cost. The Nelder and Mead downhill simplex method is applied first to find continuous duct sizes, and then a penalty approach for integer/discrete programming is applied to find discrete duct sizes. Discrete variables impose additional constraints on the design problem and the optimum cost function value is likely to increase when discrete values are assigned to variables.

Design Constraints

The design specifications are introduced as constraints in the optimization problem and the design constraints define the viability of the design solution. Tsal and Adler (1987) defined design constraints necessary for duct optimization and they are shown in the following list (constraints 1 through 8). The constraint 9 is newly added for VAV systems.

1. *Kirchhoff's first law.* The summation of the flow at each node is zero.
2. *Pressure balancing restriction.* It is required that the pressure losses be the same for all the duct paths.
3. *Nominal duct sizes.* The manufacturer sets the standard increments of duct sizes. This study follows the 1-in. (25 mm) increment for duct sizes up to 20 in. (0.5 m) and then the 2-in. (50 mm) increment.
4. *Air velocity limitation.* This is for the limiting of duct noise.
5. *Preselected sizes.* Duct sizes for some sections may be predetermined.
6. *Construction restriction.* The allowable duct sizes can be restricted for architectural reasons.
7. *Telescopic restriction.* In some systems, the diameter of the upstream duct must not be less than the diameter of the downstream duct.
8. *Standard equipment restrictions.* Duct-mounted equipment is selected from the set produced by industry.
9. *Duct static pressure control.* For a VAV system with a variable speed fan, the fan speed is often controlled to maintain a minimum static pressure at the end of the longest duct line. A minimum static pressure is required on order that no terminal unit be starved for air. To save fan energy, it is desirable that this setpoint be as low as possible. Englander and Norford (1992) suggested setpoints of 1.5 in. wg (373 Pa) for September through May and 2.5 in. wg (622 Pa) throughout the summer. These setpoints were adopted for this study.

The duct static pressure control at the end of the longest duct line directly affects system pressure loss and, accordingly, the operating cost calculation. Assuming that the fan control system exactly maintains the specified duct static pressure at the end of the longest duct line, the total fan pressure is calculated by solving the system sequentially from the terminal duct section to the root of the system. Among the above 9 constraints, constraints 1, 2, and 9 are enforced by the objective function, and constraints 5, 6, and 8 can be handled either of two ways: (1) the constraint is added to the objective function as a penalty term to provide some penalty to limit constraint violations; and (2) if the constraint states a predetermined duct size, then it can be assigned to the place of one of the variables and the number of dimensions is thereby decreased.

In this study, the duct size is used as an explicit design variable in the VAV optimization procedure. Thus, the design constraints that limit the design domain are the following:

- Nominal duct sizes
- Air velocity limitation
- Telescopic restriction

The constraint of nominal duct sizes is treated in the integer/discrete programming technique and it is introduced in the following section. Air velocity limitation sets the boundaries for duct sizes. The recommended velocities for the control of noise generation are different depending on the application; however, all categories fall within the range between 600 fpm (3 m/s) and 3,000 fpm (15 m/s) (Rowe 1988). In this study, minimum and maximum air velocity limits are set to 600 fpm (3 m/s) to 3,000 fpm (15 m/s). Telescopic restriction limits the diameter of the downstream duct. Air velocity limitation and telescopic restriction are set as penalty terms of the transformed objective function.

Penalty Function Approach for the Integer Programming

Most optimization methods have been developed under the implicit assumption that the design variables have continuous values. In many practical situations, however, the design variables are chosen from a list of commonly available values—for example, cross-section areas of trusses, thickness of plates, and the number of gear teeth. In many cases, the integer or discrete solutions are acquired by rounding the optimum continuous solution to the nearest lower or upper nominal size; however, this may often lead to an incorrect result. Thus, an effective method to find nominal duct sizes is needed in duct optimization.

Fu et al. (1991) developed a penalty function approach to solve nonlinear programming problems, including integer, discrete, and continuous variables. The approach imposes penalties of integer or discrete violation on the objective function to affect the search in the way that the solution converges to discrete standard values, based on a commonly employed optimization algorithm. In duct systems, the diameter of a round duct or the height and width of a rectangular duct is a discrete variable, and the constraint of nominal duct sizes can be resolved using the penalty function approach. Any violation of the constraint is added to the life-cycle cost to enforce the search to converge to discrete duct sizes.

In general, a discrete optimization problem can be represented as a nonlinear mathematical programming problem of the following form:

$$\text{Min} \quad f(X), \qquad X \in E^n \tag{1}$$

$$\begin{aligned} \text{Subject to:} \quad & h_i(X) = 0 && i = 1, \ldots, m \\ & G_i(X) \geq 0 && i = m+1, \ldots, p \\ & l_i \leq x_i \leq u_i \end{aligned}$$

where

$$X = [x_1, x_2, \ldots, x_n]^T = [X^c, X^d]^T$$

$X^c \in R^c$ = feasible subset of continuous design variables

$X^d \in R^d$ = feasible subset of discrete design variables

l_i and u_i = the lower and upper bounds for the design variables

The objective function may be expanded into a generalized augmented form to include penalty terms for the violation of the conditions for selecting specified discrete variable values:

$$F(X) = f(X) + P(X^d) \tag{2}$$

where $P(X^d)$ is the penalty on specified discrete value violation.

The penalty function in this approach is defined as

$$P(X^d) = \gamma Q(X^d)^{\beta} \tag{3}$$

where

$$Q(X^d) = \sum_{j \in d} 4q_j(1-q_j) \qquad \text{and} \qquad q_j = \frac{(x_j - s_j^1)}{(s_j^u - s_j^1)}; \tag{4}$$

s_j^l and s_j^u are the nearest feasible lower and upper discrete values.

Cai and Thierauf (1993) discussed the proper choice of γ and β. For β, it is recommended to choose 1 or 2. A larger value of β makes the convergence to the discrete solution slower. The choice of the value of γ strongly influences the convergence of the objective function and the following estimating equation is suggested:

$$\gamma = \frac{F(X^m) - f(X^m)}{Q^{\beta}} \tag{5}$$

where

$$X^m = (S^l + S^u)/2$$

$S^l = [s^l_{1, \ldots} s^l_n]$ and $S^u = [s^u_{1, \ldots} s^u_n]$ = the nearest lower and upper discrete points of the starting point X^0.

In the solution process, an initial value γ is estimated from the equation. When the subsequent search is made iteratively, the factor γ is gradually increased as follows:

$$\gamma^{(k+1)} = c\gamma^{(k)} \tag{6}$$

where c is a constant value in the interval, $1 < c < 2$.

The solution process for discrete programming is shown in Figure 2. Optimum continuous duct sizes from the downhill simplex method are entered as the staring point in the search for discrete duct sizes. With an estimated value of penalty factor γ, the iteration starts. For every design value, the convergence stop criterion $q_j(1 - q_j)$ is evaluated to check whether it is in a convergence limit. If it is in the limit, the discrete solution is found. The program also checks for the number of iterations. In every iteration, the penalty factor γ is gradually increased as shown in Equation 6.

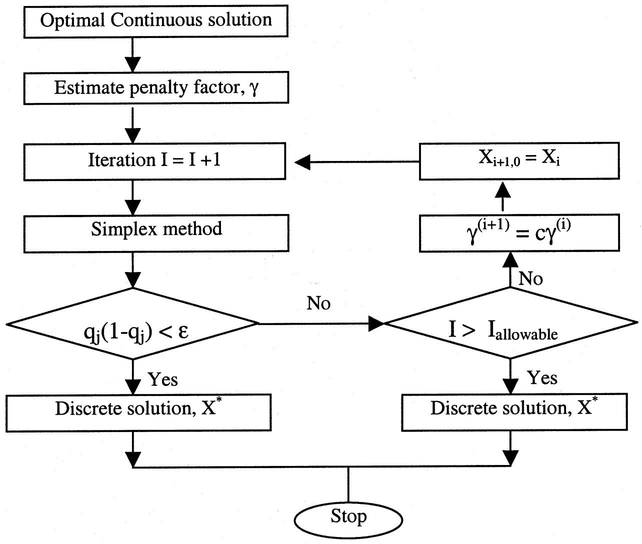


Figure 2 *Solution process of discrete programming.*

TABLE 1
Electricity Rate Structures

Site	Customer Charge	Demand Charge		Energy Charge	
		On-peak	Off-peak	On-peak	Off-peak
Tsal et al. (1988, Part II)	–	Ed: $13/kW		Ec: $0.0203/kWh	
Oklahoma	$22.8/Mo	–	–	$0.0622 k/Wh $0.0559/kWh $0.03558/kWh (June to Oct.)	$0.0373/kWh $0.0303/kWh (Nov. to May)
Minnesota	$21.65/Mo	$9.26/kW (June to Sept.)	$6.61/kW (Oct. to May)	$0.031/kWh (Jan. to Dec.)	
New York	$14.00/Mo	$11.35/kW (7 a.m. to 10 p.m.)	(10 p.m. to 7 a.m.)	$0.08755/kWh (7 a.m. to 10 p.m.)	$0.05599/kWh (10 p.m. to 7 a.m.)

ECONOMIC ANALYSIS

For a given building and duct topology, the main factors that influence optimization results are electricity costs, ductwork costs, and the VAV system operating schedule. The influence of these factors is considered by optimizing duct systems for a given building with different duct installation costs, electricity rate structures, and operating schedule.

Duct Cost

The duct materials used for optimization are as follows:

- Aluminum duct: $4.02 /ft^2 ($43.27 /m^2), (ductwork unit price used by Tsal et al. [1988]) absolute roughness 0.0001 ft (0.00003 m)
- Galvanized steel: $5.16 /ft^2 ($55.50 /m^2), (Means 2000) absolute roughness 0.0003 ft (0.00009 m)

Electric Energy Rates

As shown in Table 1, four different electric rate structures are used in this study:

- TSAL electric rate, which was introduced by Tsal et al. (1988, Part II) for Seattle, Washington
- Tulsa, Oklahoma (PSC 2000)
- Minneapolis, Minnesota (NSP 2000)
- Binghamton, New York (NYSEG 2000)

The electric rate in Tsal et al. (1988, Part II) is $0.023 / kWh for the energy charge and $13/kW for the energy demand charge without differentiating between on-peak periods and off-peak periods. In Oklahoma, the first electric rate is charged for kWh up to 150 multiplied by the current month maximum kW, the second electric rate is applied to the next 150 multiplied by the current month maximum kW, and the third electric rate is for all additional kWh used. In Minnesota, the energy demand charge is $9.26 /kW during June through September and $6.61 /kW during October through May. In New York, the on-peak period is 7 a.m. through 10 p.m., Monday through Friday.

DUCT DESIGN METHODS

In order to investigate the potential cost savings of the VAV optimization procedure, the duct design methods implemented in this study are (1) equal friction, (2) static regain, (3) T-method (Tsal et al. 1988), and (4) VAV optimization procedure.

The first two methods are commonly utilized for VAV duct design. Equal friction, static regain, and the T-method do not consider varying air volumes, so the peak airflow is used as the design air volume. The equal friction and static regain methods could generate many design solutions depending on pressure losses per foot of duct length and velocities for the duct attached to the fan, respectively. In this study, the friction rate or velocity was chosen to give the lowest life-cycle cost for one of the candidate duct systems.

When the T-method is applied to duct sizing, the fan pressure is calculated using the following equation as given in Tsal el al. (1988, Part I).

$$\Delta P_{opt} = 0.26\left(\frac{z_2}{z_1}k\right)^{0.833} + \Delta P_x \qquad (7)$$

where

$$z_1 = Q_{fan}\frac{E_c Y + E_d}{10^3 \eta_f \eta_m} PWEF, \qquad (8)$$

$$z_2 = 0.959\pi\left(\frac{\rho}{g_c}\right)^{0.2} S_d\,. \qquad (9)$$

As shown in the equation, unit energy cost E_c, energy demand cost E_d, and unit ductwork cost S_d are important factors that decide the fan pressure. The duct static pressure requirement at the end of the longest duct line is also considered in deciding fan pressure by adding that requirement to the equation. Now, based on the determined fan pressure, the T-method sizes ducts during the expansion procedure. For comparison purposes, the duct systems designed with equal friction, static regain, and the T-method are evaluated under the VAV environment to seek the life-cycle cost. The calculated costs are then compared to the one from the VAV optimization procedure to investigate the importance of the varying airflows to the system design.

EXAMPLE VAV DUCT SYSTEMS

In order to investigate the importance of varying airflows for optimum duct design, three example duct systems are selected. The duct systems are (1) ASHRAE example, (2) a large office building in Tulsa, Oklahoma, and (3) the same large office building in Minneapolis, Minnesota.

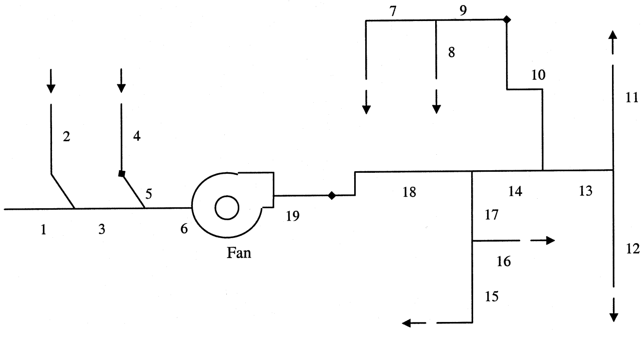


Figure 3 *ASHRAE example.*

ASHRAE Example

The ASHRAE example is a duct system given as example #3 of the *1997 ASHRAE Handbook—Fundamentals*, Chapter 32 (ASHRAE 1997). It is a 19-section duct system that has 13 supply ducts (sections 7 through 19) and 6 return ducts (sections 1 through 6). This system has been taken as a typical example in many duct design studies. The ASHRAE example in its reference is assumed to be a CAV system and the peak airflow is given to every outlet and inlet. In order to supply the systems with time-varying airflows, the fraction of full flow of the large office building in Tulsa, Oklahoma, was computed for a full year's operation using BLAST (BLAST Support Office 1986) and was used as a baseline to create varying air volumes by multiplying constant air volumes by the fraction of full flow. A schematic diagram of the system is shown in Figure 3. The sectional data of the ASHRAE example are given in Table 2. Every duct section in this example is assumed to be a round duct.

TABLE 2
Sectional Data of ASHRAE Example

Sections		Peak Airflow, cfm (m^3/s)	Duct Length, ft (m)	Additional Pres. Loss, in.wg (Pa)	ASHRAE Fitting No.*
No.	Child				
Return: 1	0, 0	1,500 (0.71)	15 (4.57)	0	ED1-3, CD9-1, ED5-1
2	0, 0	500 (0.24)	60 (18.29)	0	ED1-1,CD6-1,CD3-6,CD9-1,ED5-1
3	1, 2	2,000 (0.94)	20 (6.10)	0	CD9-1,ED5-2
4	0, 0	2,000 (0.94)	5 (1.52)	0.1 (25)	CD9-4,ER4-3
5	4, 0	2,000 (0.94)	55 (16.76)	0	CD3-17,CD9-1,ED5-2
6	3, 5	4,000 (1.89)	30 (9.14)	0.22 (55)	CD9-3,CD3-9,ED7-2
Supply: 7	0, 0	600 (0.28)	14 (4.27)	0.1 (25)	CR3-3,CR9-1,SR5-13
8	0, 0	600 (0.28)	4 (1.22)	0.15 (37)	SR5-13,CR9-4
9	7, 8	1,200 (0.57)	25 (7.62)	0	SR3-1
10	9, 0	1,200 (0.57)	45 (13.72)	0	CR9-1,CR3-10,CR3-6,SR5-1
11	0, 0	1,000 (0.47)	10 (3.05)	0	CR9-1,SR2-1,SR5-14
12	0, 0	1,000 (0.47)	22 (6.71)	0	CR9-1,SR2-5,SR5-14
13	11, 12	2,000 (0.94)	35 (10.67)	0	CR9-1,SR5-1
14	10, 13	3,200 (1.51)	15 (4.57)	0	CR9-1,SR5-13
15	0, 0	400 (0.19)	40 (12.19)	0	CR3-1,SR2-6,CR9-1,SR5-1
16	0, 0	400 (0.19)	20 (6.10)	0	SR2-3,CR6-1,CR9-1,SR5-1
17	15, 16	800 (0.38)	22 (6.71)	0	CR9-1,SR5-13
18	14, 17	4,000 (1.89)	23 (7.01)	0.04 (10)	CR6-4,SR4-1,CR3-17,CR9-6
Root: 19	18, 0	4,000 (1.89)	12 (3.66)	0.05 (13)	SR7-17,CR9-4

* From ASHRAE Duct Fitting Database (1993)

Large Office Building in Oklahoma and Minnesota

The large office building in Oklahoma is a 34-section supply duct system of part of a single floor of the BOK building in Tulsa. The building is a 52-story multipurpose office building located in Tulsa's downtown area. It measures about 160 ft (48.77 m) by 160 ft (48.77 m) and is about 1,360 ft (414.53 m) in height. The building is oriented in a 20° northeast north direction and is not shaded by any other structures. It has a large area of glazing—about 65% of the exterior envelope. The building is described by Feng (1999) in greater detail. The example duct system from this building serves only part of a single floor—zones 18 through 22 as shown in Figure 4, approximately 13,200 ft^2 (1226 m^2) of floor space. A schematic diagram with section numbers is shown in Figure 5. The sectional data of the VAV duct system are given in Table 3. The air-handling unit is located at Zone 19 and air is distributed to perimeter zones 20, 21, and 22. Zone 20 on the east side has two terminal boxes and eight exits, zone 21 on the north side has four terminal boxes and fifteen exits, and zone 22 has two terminal boxes and six exits. Every duct section is assumed to be a round duct.

The HVAC system for this floor was originally a three-deck multizone system that featured hot and cold decks and separate mixing dampers for each zone. In this study, it is assumed to have a VAV system in which air flows through a main cooling coil at a design cold-deck temperature of 55ºF (12.78ºC). The air is then sent to each zone by modulating the amount of air with a VAV box. If the zone requires heating, the air is heated by use of auxiliary reheat. The system has a VAV control schedule that specifies the fraction of peak cooling or heating at a specific zone temperature for a VAV system. For this study, the occupancy, lighting, and equipment profiles for the building are assumed to have a weekday schedule of being fully on from 8 a.m. to 5 p.m., and the building is assumed to have no occupancy, lighting, or equipment heat gain for nights, weekends, and holidays. The system was simulated based on two different operating schedules: (1) 8760-hour schedule (always on) and (2) setback controlled schedule. The 8760-hour schedule has VAV control for 24 hours a day, all year long, while the setback controlled schedule has VAV control from 7 a.m. to 5 p.m., Monday through Friday, and setback control from 5 p.m. to 7 a.m., Monday through Friday, and all day Saturday, Sunday, and holidays. The setback controlled schedule results in 2,763 hours of operation for the large office building in Oklahoma. All the VAV boxes have minimum fractions of 0.4. Airflow data summed for all zones are represented using a histogram, which is a frequency distribution with the fraction of full flow as the abscissa and the number of hours at each increment as the ordinate in Figure 6. In the figure, bin 1 corresponds to 0% to 5% of full flow and bin 2 corresponds to 5% to 10% of full flow, etc. In Figure 6a, for the large office building in Oklahoma, bin 9, which corresponds to the minimum fraction of full flow, has 6,623 operating hours for the 8,760-hour schedule. In Figure 6b, the setback controlled schedule results in 2,763 hours operation, of which 1,288 hours are at the of minimum fraction.

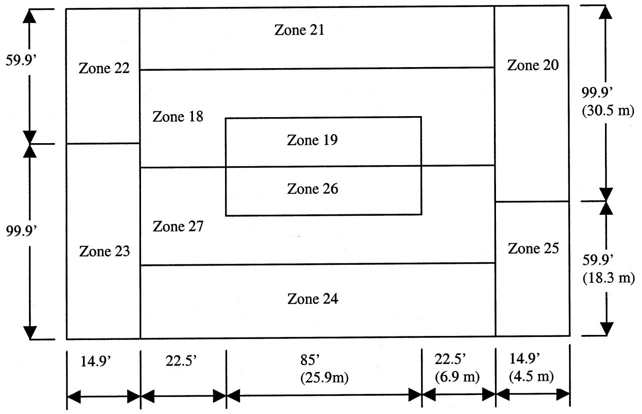


Figure 4 *Zone layout for floors 8 through 24 of the large office building.*

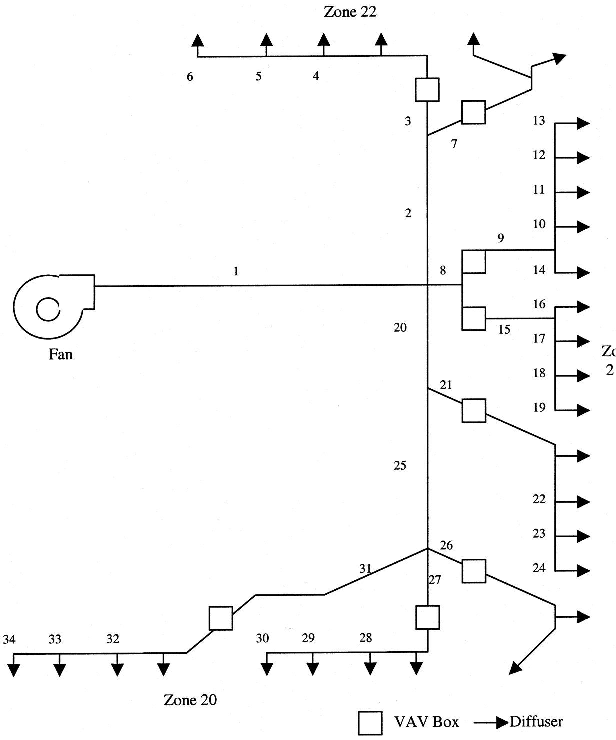


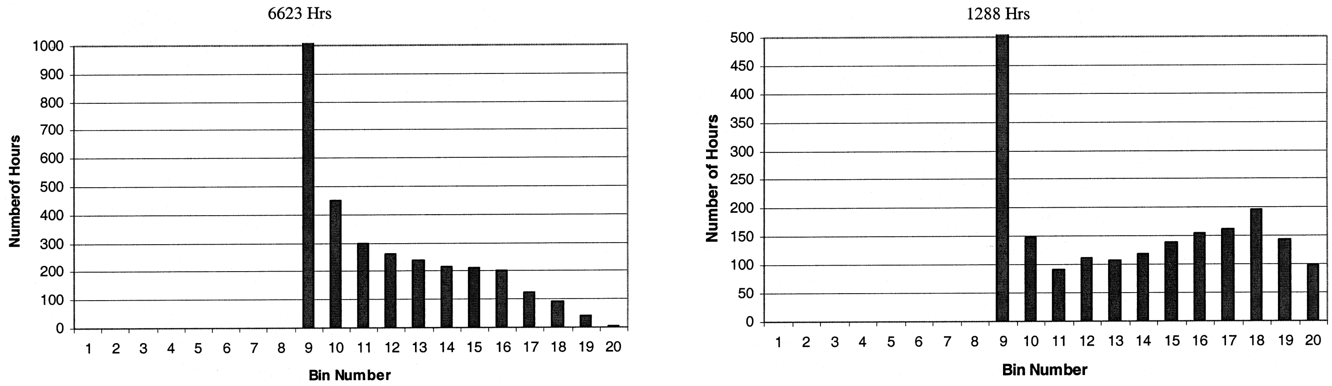
Figure 5 *Schematic diagram of the duct system of the large office building.*

The large office building in Minnesota shares the same layout and sectional information as the one at Oklahoma. The building is simulated at Minneapolis, Minnesota, in order to investigate the effect of climate on optimum duct design with different weather conditions. All duct sections are again assumed to be round ducts. The histogram of airflow data of the building in Minnesota is shown in Figure 7. In Figure 7(a), bin 9, which corresponds to the minimum fraction of full flow,

TABLE 3
Sectional Data of the Large Office Building in Oklahoma and Minnesota

Section		Peak Airflow, cfm (m^3/s)		Duct Length, ft (m)	DPz, in.wg (Pa)	ASHRAE Fitting No.*
No.	Child	Tulsa, Okla.	Minneapolis, Minn.			
1	2, 8, 19	8,678 (4.095)	7,584 (3.579)	50 (15.24)	0 (0)	SR7-17, SD4-2, CD3-9,
2	3, 7	2,699 (1.274)	2,481 (1.171)	35 (10.67)	0 (0)	SD5-26(b1),
3	4	1,800 (0.849)	1,654 (0.780)	25 (7.62)	0.2 (50)	SD5-2(S), CD3-8, CD3-8, CD9-1, CD3-9,SD4-1
4	5	1,350 (0.637)	1,240 (0.585)	10 (3.05)	0.2 (50)	SD4-1
5	6	900 (0.425)	827 (0.390)	10 (3.05)	0.2 (50)	SD4-1
6	-	450 (0.212)	413 (0.195)	10 (3.05)	0.2 (50)	-
7	-	900 (0.425)	827 (0.390)	20 (6.10)	0.4 (100)	SD5-2(b),CD9-1,CD3-14
8	9, 14	1,637 (0.773)	1,254 (0.592)	5 (1.52)	0 (0)	SD5-26(s)
9	10, 13	910 (0.429)	697 (0.329)	15 (4.57)	0 (0)	SD5-19,CD9-1,CD3-9
10	11	728 (0.343)	558 (0.263)	10 (3.05)	0.2 (50)	SD5-19(b1), SD4-1
11	12	546 (0.258)	418 (0.197)	10 (3.05)	0.2 (50)	SD4-1
12	13	364 (0.172)	279 (0.132)	10 (3.05)	0.2 (50)	SD4-1
13	-	182 (0.086)	139 (0.066)	10 (3.05)	0.2 (50)	-
14	-	182 (0.086)	139 (0.066)	10 (3.05)	0.2 (50)	SD5-19(b2)
15	16, 17	728 (0.343)	558 (0.263)	15 (4.57)	0 (0)	SD5-19,CD9-1,CD3-9
16	-	182 (0.086)	139 (0.066)	10 (3.05)	0.2 (50)	SD5-19(b1)
17	18	546 (0.258)	418 (0.197)	10 (3.05)	0.2 (50)	SD5-19(b1), SD4-1
18	19	364 (0.172)	279 (0.132)	10 (3.05)	0.2 (50)	SD4-1
19	-	182 (0.086)	139 (0.066)	10 (3.05)	0.2 (50)	-
20	21, 25	4,341 (2.049)	3,849 (1.816)	30 (9.14)	0 (0)	SD5-26(b2)
21	22	728 (0.343)	558 (0.263)	20 (6.10)	0.2 (50)	SD5-2(b),CD9-1,CD3-14,SD4-1
22	23	546 (0.258)	418 (0.197)	10 (3.05)	0.2 (50)	SD4-1
23	24	364 (0.172)	279 (0.132)	10 (3.05)	0.2 (50)	SD4-1
24	-	182 (0.086)	139 (0.066)	10 (3.05)	0.2 (50)	-
25	26,27, 31	3,614 (1.705)	3,291 (1.553)	40 (12.19)	0 (0)	SD5-2(s)
26	-	723 (0.341)	658 (0.311)	20 (6.10)	0.4 (100)	SD5-23(b1), CD9-1, CD3-14,
27	28	1,445 (0.682)	1,317 (0.621)	25 (7.62)	0.2 (50)	SD5-23(S),CD9-1,CD3-9
28	29	1,084 (0.512)	987 (0.466)	10 (3.05)	0.2 (50)	SD4-1
29	30	723 (0.341)	658 (0.311)	10 (3.05)	0.2 (50)	SD4-1
30		361 (0.171)	329 (0.155)	10 (3.05)	0.2 (50)	-
31	32	1,445 (0.682)	1,317 (0.621)	60 (18.29)	0.2 (50)	SD5-23(b2), CD9-1, CD3-14,CD3-14, CD3-14,
32	33	1,084 (0.512)	987 (0.466)	10 (3.05)	0.2 (50)	SD4-1
33	34	723 (0.341)	658 (0.311)	10 (3.05)	0.2 (50)	SD4-1
34	-	361 (0.171)	329 (0.155)	10 (3.05)	0.2 (50)	-

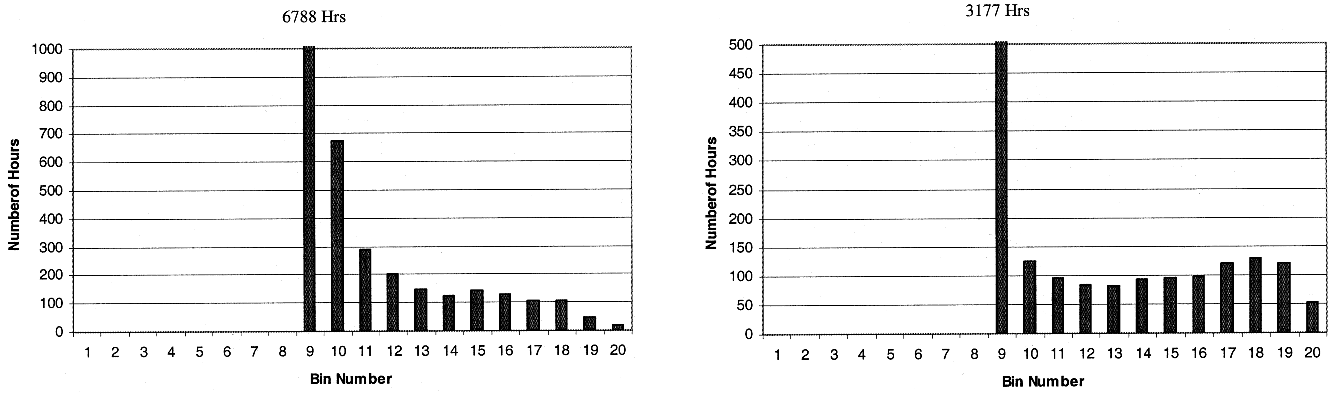
[*] From ASHRAE Duct Fitting Database (1993)



(a) 8760-hour schedule

(b) Setback controlled schedule: 2763 hours

Figure 6 *Annual distribution of fraction of full flow of the large office building in Oklahoma.*



(a) 8760-hour schedule

(b) Setback controlled schedule: 4269 hours

Figure 7 *Annual distribution of fraction of full flow of the large office building in Minnesota.*

has 6,788 operating hours for the 8,760-hour schedule. In Figure 7b, the setback controlled schedule results in 4,269 hours of operation and 3,177 hours of minimum fraction of full flow.

COMPUTATION RESULTS

The three duct systems are sized using four different design methods for four different electric rates and two different operating schedules with aluminum ducts and galvanized steel ducts. Duct sizes from the equal friction method are obtained with the pressure loss per 100 ft that gives the lowest life-cycle cost with the TSAL electric rate, 0.15 in. wg/100 ft (1.22 Pa/m) for the ASHRAE example, 0.15 in. wg/100 ft (1.22 Pa/m) for the office building in Oklahoma, and 0.1 in. wg/100 ft (0.82 Pa/m) for the office building in Minnesota. Duct sizes from the static regain method are obtained with the velocity of the duct attached to the fan that gives the lowest life-cycle cost with the TSAL electric rate, 2,600 fpm (13 m/s) for the ASHRAE example, 2,800 fpm (14 m/s) for the office building in Oklahoma, and 2,700 fpm (13.5 m/s) for the office building in Minnesota. The duct systems designed with the equal friction and static regain methods are then simulated with different electric rates in order to see the economic effect under VAV operation. In the T-method, different electric rates establish different optimum duct sizes since the optimum fan pressure is changed. The VAV optimization procedure established optimum duct sizes with varying airflows through the selection of an efficient fan, finding duct fitting coefficients, calculating system pressure loss, and evaluating the life-cycle cost. Typically, 20,000 to 25,000 objective function evaluations are utilized. For duct size rounding or discrete programming, a 1-in. increment is used for duct sizes up to 20 in. and a 2-in. increment is used for all others. The results are organized as follows:

- Comparison of duct design methods: life-cycle cost analysis, duct surface area.
- Influences on the optimal design: effect of electric rate on optimal design, effect of ductwork unit cost on optimal design, effect of topology on optimal design, effect of airflow schedules, unconstrained optimization results, optimization domain.

TABLE 4
Life-Cycle Cost and Savings of the VAV Optimization Procedure (Aluminum Ducts)*
(Unit:$)

Duct System	Duct Design Method	TSAL Electric Rate	Okla. Electric Rate	Minn. Electric Rate	N.Y. Electric Rate
ASHRAE example	Equal friction	13,263 (14.4%)	17,293 (9.4%)	17,664 (9.2%)	21,424 (6.1%)
	ASHRAE ans.	12,125 (6.4%)	16,231 (3.5%)	16,640 (3.6%)	20,560 (2.2%)
	Static regain	12,270 (7.5%)	16,407 (4.5%)	16,408 (2.2%)	20,837 (3.5%)
	T-method	11,665 (2.7%)	15,960 (1.8%)	16,377 (2.0%)	20,547 (2.1%)
	VAV opt.	11,348	15,668	16,044	20,115
Building in Okla.	Equal friction	14,470 (17.3%)	19,223 (12.2%)	20,010 (11.5%)	25,718 (7.8%)
	Static regain	13,303 (10.0%)	18,165 (7.0%)	19,025 (6.9%)	25,060 (5.3%)
	T-method	12,227 (2.1%)	17,093 (1.2%)	17,972 (1.5%)	24,042 (1.3%)
	VAV opt.	11,967	16,887	17,707	23,720
Building in Minn.	Equal friction	13,419 (18.7%)	17,777 (12.2%)	18,536 (11.7%)	23,246 (8.7%)
	Static regain	12,571 (13.3%)	16,989 (8.1%)	17,790 (8.0%)	22,673 (6.4%)
	T-method	11,370 (4.1%)	15,806 (1.2%)	16,609 (1.4%)	21,617 (1.8%)
	VAV opt.	10,905	15,617	16,369	21,231

* All costs are listed in Tables A7 and A8 of the appendix.

COMPARISON OF DUCT DESIGN METHODS

Three duct systems designed with four different design methods were evaluated under VAV operation for cost comparison purposes. The evaluation generated the initial, operating, and life-cycle costs, and they were compared to investigate the savings of the VAV optimization procedure. Furthermore, duct designs were compared for four different electric rate structures, although only the VAV optimization procedure and the T-method take the electricity rate into account in the duct design.

Life-Cycle Cost Analysis. Table 4 shows the life-cycle cost when the three duct systems are designed under four different electricity rate structures using aluminum ducts. The percentage saved by the VAV optimization procedure compared to the other design methods is shown in parenthesis. When the life-cycle cost of the VAV optimization procedure is compared with the other duct design methods as shown in Table 4, the VAV optimization procedure shows 6% to 19% savings for the equal friction method, 2% to 13% savings over the static regain method, and 1% to 4% savings over the T-method.

The VAV optimization procedure gives greater life-cycle cost savings with lower electricity rates. For example, the ASHRAE example shows that the savings with the TSAL electric rate compared to the equal friction, static regain, and T-method are 14%, 8%, and 3%; and the savings with the N.Y. electric rate are 6%, 4%, and 2%, respectively. The better savings with lower electric rates are explained below in the section, “Optimization Domain.” While the VAV optimization procedure gives larger savings for the large office building for all electricity rates, the savings are similarly lower for higher electricity rates.

TABLE 5
Life-Cycle Cost and Savings of the VAV Optimization Procedure (Galvanized Steel Ducts)*
(Unit: $)

Duct System	Duct Design Method	Okla. Electric Rate	Minn. Electric Rate	N.Y. Electric Rate
Building in Okla.	Equal friction	21,893 (12.7%)	22,720 (11.0%)	28,606 (6.8%)
	Static regain	20,712 (7.7%)	21,641 (6.6%)	27,996 (4.8%)
	T-method	19,276 (0.8%)	20,292 (0.4%)	26,831 (0.7%)
	VAV opt.	19,123	20,213	26,654

* Optimum duct sizes and economic costs are shown in Tables A4 and A9 of the appendix

Table 5 shows the life-cycle cost and savings when galvanized steel, which has a slightly higher unit cost than the aluminum ducts, is used for ducts. The VAV optimization procedure yields life-cycle cost savings ranging from 6.8% to 12.7% over the equal friction method, 4.8% to 7.7% over the static regain method, and 0.4% to 0.8% over the T-method. This is a slight decrease in savings compared to aluminum ducts. The aluminum duct system designed with the VAV optimization procedure was nearly completely constrained by the lower limits on duct size, so that little reduction in duct size was possible with the more expensive galvanized steel ducts. Therefore, for this building, the VAV optimization procedure's performance, relative to the T-method, decreases as duct unit cost increases.

Duct Surface Area. Table 6 gives a comparison of the duct surface area of three duct systems using different duct design methods. The VAV optimization procedure saved duct surface by 23% to 31% over the equal friction method, 13% to 22% over the static regain method, and 4% to 7% over the T-method.

A characteristic of the problem is that the duct system has an optimum solution near to the minimum sizes, depending on the electric rate. For the large office building in Oklahoma and Minnesota, the duct system is almost completely constrained to the lower limit duct sizes with the TSAL electric rate. However, when the higher electric rate is used, optimum duct sizes for most duct sections are higher than the lower bound. For the ASHRAE example, some optimum duct sizes are coincident and others are near to the lower limits with the TSAL electric rate. This differs from the nearly completely constrained large office building. The reason for this different optimal solution location depends on the problem is discussed later in the section, "Effect of Topology on Optimal Design."

TABLE 6
Comparison of Duct Surfaces Using Different Duct Design Methods (Aluminum Ducts)*

Duct System	Duct Surface, ft^2 (m^2)			
	Equal Friction	Static Regain	T-method	VAV Opt. Proc.
ASHRAE Ex. with TSAL E. rate	2,271 (211)	1,996 (185)	1,824 (169)	1,740 (162)
Building at Okla. with Okla. E. rate	2,225 (207)	1,897 (176)	1,645 (153)	1,585 (147)
Building at Minn. with Minn. E. rate	2,100 (195)	1,868 (174)	1,564 (145)	1,458 (135)

* Optimum duct sizes are shown in Tables A1 to A3 of the appendix.

Influences on the Optimal Design

There are several factors that influence optimal duct design—electricity rate, ductwork unit cost, duct topology, airflow schedule, and constraints (air velocity/duct diameter). The influences of these factors are investigated and discussed below.

Effect of Electricity Rate on Optimal Design. Table 7 shows the optimal duct surface area found with different electric rate structures using the VAV optimization procedure. The electricity cost of New York ($0.08755 /kWh) is about four times higher than that of the TSAL electric rate ($0.0203 / kWh), while having a similar demand charge. It might be expected that the higher electricity rates would cause the duct system diameters in New York to significantly increase in order to lower the operating cost. But, in fact, the duct surface with the N.Y. electric rate is only 4.2% to 6.7% higher than that with the TSAL electric rate as shown in Table 7.

The increase of operating cost due to higher electricity rates should be offset by an optimal design that has larger duct sizes and, hence, larger initial cost, but also lower system pressure drop and lower operating cost. But, in fact, the increased duct diameters make a small impact on the average total system pressure drop because of the requirement to maintain a fixed static pressure at the end of the longest duct line. This static pressure requirement limits the potential reduction of the operating cost. For example, when the optimal duct diameters of the large office building in Oklahoma with the N.Y. electric rate is doubled (duct surface area changed from 1,614 ft^2 [150 m^2] to 3,228 ft^2 [300 m^2]), it is expected that the system pressure drop should be reduced by a factor of 2^5 or the average pressure drop should only be about 3% of the original system. However, the average total pressure drop, including the 1.5 in. static pressure requirement, is only reduced by 12%. The average system pressure drop for 8,760 h was 1.721 in. wg without doubled duct sizes and 1.514 in. wg with doubled duct sizes. The system pressure drop at minimum airflow is 1.646 in. wg without doubled duct sizes and 1.507 in. wg with doubled duct sizes. At full flow, the system pressure drop is 2.787 in. wg without doubled duct sizes and 1.533 in. wg with doubled duct sizes. At minimum airflow, the static pressure requirement at the end of the longest duct line domi-

TABLE 7
Comparison of Duct Surfaces with Different Electric Rate Structures*

Duct System	Duct Surface, ft^2 (m^2)				Surface Increase% (TSAL to N.Y.)
	TSAL Electric Rate	Okla. Electric Rate	Minn. Electric Rate	N.Y. Electric Rate	
ASHRAE ex.	1,740 (162)	1,768 (164)	1,773 (165)	1,856 (172)	6.7
Building in Okla.	1,546 (144)	1,585 (147)	1,551 (144)	1,611 (150)	4.2
Building in Minn.	1,410 (131)	1,496 (139)	1,458 (136)	1,521 (141)	7.9

* Optimum duct sizes are shown in Tables A1 to A3 of the appendix.

TABLE 8
Optimization for Different Ductwork Unit Costs (Galvanized Steel Ducts)

Ductwork Unit Cost	Duct Surface, ft^2 (m^2)	Initial Cost, $	Opr. Cost, $	L.C. Cost, $
Aluminum ducts, $4.02 /ft^2 ($43.27 /m^2)	1,585 (147.3)	8,947	7,940	16,887
Galvanized steel ducts, $5.16 /ft^2 ($55.50 /m^2)	1,543 (143.4)	10,532	8,591	19,123
Doubled cost, $10.31 /ft^2 ($111.00 /m^2)	1,530 (142.2)	18,355	8,674	27,029

TABLE 9
Optimization for Different System Operation Schedules*

Duct System	Operation Schedule	Duct Surface, ft^2(m^2)	Initial Cost, $	Opr. Cost, $	L.C. Cost, $
Building in Okla. with Okla. E rate	8,760-hr	1,585 (147)	8,947	7,940	16,887
	Setback control	1,614 (150)	9,063	5,845	14,908
Building in Minn. with Minn. E rate	8,760-hr	1,458 (136)	8,282	8,087	16,369
	Setback control	1,480 (138)	8,371	7,182	15,554

* Duct sizes and costs for setback controlled schedule are shown in Tables A5, A6, and A10 of the appendix.

nates system pressure loss since the pressure losses in the ducts are very low. Considering the VAV system is operated much of time at lower airflows, the system pressure losses for a full year's operation do not change greatly with the change in duct sizes. Consequently, the change of operating costs becomes small and does not force a significant change in duct sizes.

Effect of Ductwork Unit Cost on Optimal Design. Table 8 shows the optimal duct surface area and operating cost found with the VAV optimization for the large office building in Oklahoma with the Oklahoma electric rate. The ductwork costs are $4.02 /ft^2 ($43.27 /m^2) for aluminum ducts and $5.16 /ft^2 ($55.50 /m^2) for galvanized steel ducts, and to investigate sensitivity a "double" ductwork unit cost of $10.31 /ft^2 ($111.00 /m^2) was set. It might be expected that the higher ductwork unit cost would cause the ducts to be sized smaller, which, in turn, would give a higher operating cost but a lower initial cost. However, the optimal duct surface area with a double ductwork unit cost is reduced only a little with a small increase of the operating cost as shown in Table 8.

The small reduction of the optimal duct surface area results from the velocity/minimum duct size constraint. The optimization with a double ductwork unit cost could not make a further reduction in many duct sections since the diameters of the original duct system are coincident and near to the lower limits, which are constrained by the upper velocity limitation (See Table A4 in Appendix for the change of individual duct sizes of galvanized steel ducts).

Effect of Topology on Optimal Design. In an earlier section, it was noted that the large office building example was almost completely constrained to the lower limit duct sizes with the TSAL electric rate, while the ASHRAE example is constrained to the lower limit only for a few duct sections. They had the same electric rate, ductwork cost, and similar airflow distributions, but the result was different. A possible explanation is the difference in duct topology. In order to investigate this difference, the large office building in Oklahoma with the TSAL electric rate is optimized again after increasing all duct lengths by 20% and 40%. The result was that the enlarged duct system moved duct sections off the constraint. The original duct system had 5 of 34 duct sections not on the minimum size constraint. When the duct lengths were increased by 20%, 9 of 34 duct sections were not on the constraint. When the duct lengths were increased by 40%, two duct diameters increased—one duct section was moved off the constraint and another duct section increased duct diameter by 2 in. This indicates that the VAV optimization procedure finds optimal solution near the lower limit duct sizes with the low electric rate, but the number of duct sections at the constraint depends on partly the duct topology.

Effect of Airflow Schedules. The climatic conditions and the system operation schedule both affect the hourly distribution of airflow rates. These have a relatively minor effect on the optimal duct sizes.

Climatic condition: The large office building in Tulsa, Oklahoma, was optimized under two different weather conditions: (1) Oklahoma and (2) Minnesota. Two different airflow data sets were created, including two different system capacities and peak airflow rates, for the same duct topology. For the four different electric rates, the life-cycle cost of the large office building duct system in Minnesota is 8% to 10% lower than that of the building in Oklahoma. The duct cost of the building in Minnesota is 6% to 10% lower than that of the building in Oklahoma. As expected, the duct system in a cold climate has a smaller duct system with a saving in the life-cycle cost.

System operation schedule: Table 9 shows the computation results for two different system operation schedules using

TABLE 10
Unconstrained Optimization of the Large Office Building in Oklahoma with No Velocity Limitation and Zero Static Pressure Requirement (Aluminum Ducts)

Electric Rate		Duct Surface, ft^2 (m^2)	Initial Cost, $	Opr. Cost, $	L.C. Cost, $	Saving%
TSAL	VAV opt. proc.	1,643 (152.6)	9,179	2,860	12,039	-
	T-method	2,127 (197.6)	11,126	2,341	13,467	10.6
N.Y.	VAV opt. proc.	1,958 (181.9)	10,447	11,919	22,366	-
	T-method	2,775 (252.5)	13,499	11,019	24,518	8.8

the VAV optimization procedure. The setback controlled schedule results in 2,763 hours of operation for the large office building in Oklahoma and 4,269 hours operation for the large office building in Minnesota. Comparing optimal duct areas, it is expected that the duct system with the setback controlled operation should have smaller ducts compared to the one with the 8,760-hour operation, as with a smaller number of operating hours, the effect of the operating cost on the life-cycle cost should be less. However, as shown in Table 9, the system with the setback controlled operation has 1.5% to 1.8% larger optimal duct surface area. The setback controlled operation requires a larger peak airflow than the 8,760-hour operation due to morning start-up loads. The larger airflow results in higher system pressure drop and higher operating cost. Hence, the increase of operating cost is offset by an optimal design that has slightly larger duct sizes.

Unconstrained Optimization Results. The comparison of duct design methods presented above showed that the VAV optimization procedure did not give significantly better results than the T-method. Further investigation of the effects of electricity rates, ductwork unit costs, topology, and airflow schedules led to the observation that with the test buildings and electricity rates used,

- the problem tends to be constrained by the maximum velocity/minimum duct diameter;
- the minimum static pressure requirements lead to a rapidly diminishing point of return—operating costs can only be reduced up to a point by increasing the duct size.

In order to confirm these observations, a rather artificial comparison is performed. Table 10 shows an unconstrained optimization of the large office building duct system in Oklahoma with the New York electric rate and zero in. wg static pressure requirement.

The life-cycle cost savings of the VAV optimization procedure increased from 2.1% to 10.6% for the TSAL electric rate and 1.3% to 8.8% for the New York electric rate. The VAV optimization procedure gives much better savings with a lower electric rate, no size constraints, and no static pressure requirement. This provides some confirmation for the above observations. The static pressure requirement and velocity constraints prevent the VAV optimization procedure from finding significantly better designs.

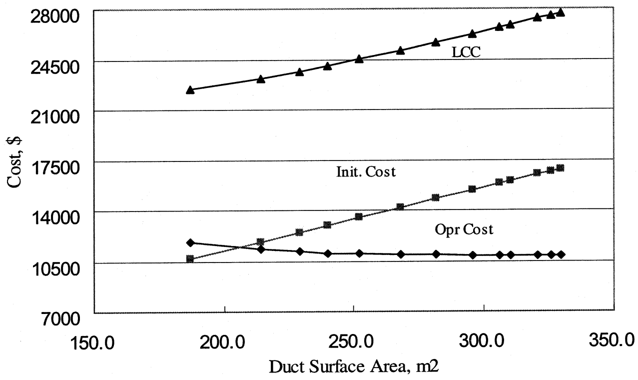


Figure 8 *Optimization domain of duct systems of the large office building in Oklahoma with N.Y. electric rate.*

Optimization Domain. Over the course of this investigation, it has been observed that the life-cycle cost does not seem to be as sensitive to the duct design as originally expected. Although the optimization domain is not relatively flat when viewed as a function of individual duct sizes, there is a sense in which, if the individual ducts are correctly sized relative to one another, the domain is relatively "flat" (i.e., the life-cycle cost is relatively insensitive to the total duct surface area). To help explain this, consider the following. If the T-method is utilized to size duct systems for the large office building in Oklahoma with no velocity limitation and no static pressure requirements but with a range of electricity rates, a corresponding range of duct systems will result. The life-cycle cost for these system are calculated using a fixed electricity rate.

Figure 8 is a representation of the optimization domain for the large office building in Oklahoma with the New York electric rate. Economic costs are plotted in terms of total duct surface area. Each point represents a duct system that has been optimized with the T-method for an electric rate that is higher or lower than the actual N.Y. electric rate. However, the operating costs are calculated with the actual N.Y. electric rate. From the figure, the life-cycle cost has a gently increasing slope, demonstrating that the life-cycle cost is relatively insensitive to the total duct surface area. If the static pressure requirement is included in the duct design, the curve will be much flatter and the life-cycle cost will be more insensitive to the total duct surface area. The life-cycle cost savings of the VAV optimization procedure relative to the T-method will be further lowered. When the curves for initial and operating costs are compared, the initial cost curve is steeper than the operating cost curve.

Figure 9 is the plot of economic costs in terms of the electric rate multiplier. Both the kWh charge and the demand charge for New York were multiplied by the electric rate multiplier. Again, at each point, the operating cost was calculated with the actual electric rate. When finding a design solution, the T-method uses the peak airflow and the VAV optimization uses a range of airflows, but they are dominated by the minimum airflows. This, in turn, results in the VAV optimization procedure calculating a lower operating cost for any given duct configuration. The savings in life-cycle cost yielded by the VAV optimization procedure result from it being able to take advantage of the knowledge that the operating cost is lower than that calculated by the T-method. Although this is due to using the actual flow rates, it is analogous to having a lower electricity rate for the VAV optimization procedure. For any given building/duct topology/climate, etc., combination, the reduced operating cost is equivalent to a fixed percentage reduction in the electricity rate. As can be seen in Figure 9, the operating cost, initial cost, and life-cycle cost all change more rapidly at lower electricity rates. Arguing by analogy provides an explanation for why the VAV optimization procedure (compared to the T-method) performs better at lower electricity rates. At lower electricity rates, the life-cycle cost is more sensitive to a change in electricity rate or a change in operating cost caused by evaluating electricity consumption at low flow rates.

However, in actual practice, the change in performance is significantly damped by duct size constraints and static pressure requirements. Therefore, the VAV optimization procedure does not appear to offer significant enough savings to warrant its use in practice, since it requires a significant increase in the amount of input data and the computational time required. Instead, the T-method seems to offer a good balance between results and ease-of-use when implemented in a computer program.

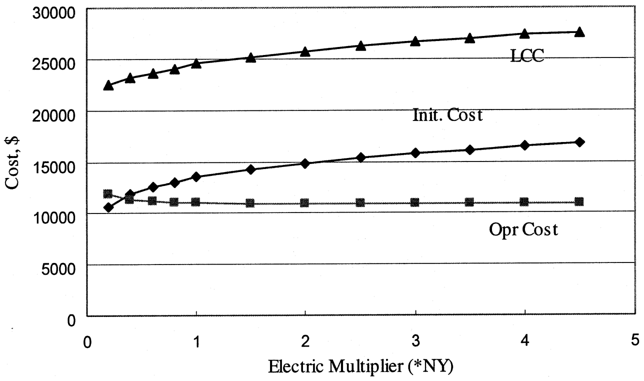


Figure 9 *Plot of costs in terms of electric rate of duct systems of the large office building in Oklahoma with the N.Y. electric rate.*

CONCLUSIONS

The VAV optimization procedure was applied to the three VAV duct systems to investigate the impact of varying airflow rates on the sizing of duct systems. For comparison purposes, other duct design methods, such as equal friction, static regain, and the T-method, were also applied to the duct systems. While the VAV optimization procedure uses varying airflows, the other methods use peak constant airflows for duct system design. The equal friction and static regain methods calculate system pressure loss after duct sizes are decided. The T-method calculated fan pressure using electric rate and ductwork unit cost and then optimized duct sizes. The VAV optimization procedure selects a fan by checking as to whether the system design point with the peak hour's airflow and other system operating points for varying airflows reside in the fan operating region. After fan selection, the downhill simplex method searches optimum duct sizes through evaluation of the life-cycle cost.

After optimum duct sizes are found, the duct systems resulting from four different duct design methods are simulated under operation for a typical meteorological year in order to investigate the performance of the methods. With respect to life-cycle cost, the VAV optimization procedure showed 6% to 19% savings compared to the equal friction method, 2% to 13% savings over the static regain method, and 1% to 4% savings over the T-method. Compared to the T-method, the VAV optimization procedure gives a lower initial cost and a higher operating cost. The total duct surface, and, hence, the initial cost, using the VAV optimization procedure was significantly lower compared to other duct design methods. The VAV optimization procedure saved duct surface 23% to 31% over the equal friction method, 13% to 22% over the static regain method, and 4% to 7% over the T-method.

Trends that were identified include:

- The VAV optimization procedure allowed greater life-cycle cost savings (compared to the T-method) with lower electricity rates.
- For the large office building, the life-cycle cost savings of the VAV optimization procedure compared to the T-method decrease as duct unit cost increases.
- The duct topology influences the degree to which the optimal solution is at the duct size constraints. Longer duct lengths tended to reduce the number of duct sections at the minimum size constraint.
- While different climatic conditions and operating schedules influenced the optimal design, they did not have a significant impact on the savings of the VAV optimization procedure compared to the T-method.

In general, the VAV optimization procedure yields significant life-cycle cost savings compared with the equal friction and static regain methods. However, compared with the T-method, the life-cycle cost savings of the VAV optimization procedure was not as great. This is partly due to two significant limitations that prevent the VAV optimization procedure from finding significantly better designs. First, optimal duct sizes are found near to the lower limits, which are constrained by the velocity limitation. Second, the change of system pressure drop due to changing the duct surface area is smaller than expected because of the static pressure requirement at the longest duct line.

Even when these limitations are artificially eliminated, the optimization domain analysis showed that the life-cycle cost is relatively insensitive to the total duct surface area when the duct design has been arrived at by the T-method. This indicates that the T-method can be used favorably even in VAV system optimization. The T-method has great potential to save costs over the non-optimization-based methods, without the input data and computation time requirements of the VAV optimization method. Therefore, it is recommended that the T-method be utilized for duct design in VAV systems. Given the marginal improvement in life-cycle cost yielded by the VAV optimization procedure compared to the T-method, further research is probably not warranted at this time. Nevertheless, if the situation arose where the procedure could be profitably applied, it would be useful to decrease the computational requirements. This might be achieved by modeling a statistical representation of the airflow data rather than all 8,760 hours.

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APPENDIX A

TABLE A-1
Duct Sizes of the ASHRAE Example (8760-Hour Schedule, Aluminum Ducts)*
Unit: in. (1 in. = 0.254 m)

Duct section	Max. Airflow, cfm	Min. Duct Size	Max. Duct Size	Equal Friction	ASHRAE Ans.	Static Regain	T-method				VAV Optimization Procedure			
							TSAL E Rate	Okla. E Rate	Minn. E Rate	N.y. E Rate	TSAL E Rate	Okla. E Rate	Minn. E Rate	N.y. E Rate
Return: 1	1,504	10	20	15	12.00	14	10	11	11	11	10	10	11	12
2	509	6	12	10	8.00	9	7	7	7	8	7	7	7	8
3	2,013	11	24	17	12.00	14	13	14	14	14	13	14	14	13
4	1,992	11	24	17	26.20	14	16	17	17	18	12	12	13	14
5	1,992	11	24	17	15.00	14	12	13	13	13	12	13	13	14
6	4,005	16	34	22	17.00	17	20	20	20	22	17	17	17	18
Supply:7	593	6	13	11	10.90	9	8	8	8	8	6	6	6	6
8	593	6	13	11	10.90	9	8	8	10	9	12	13	13	13
9	1,187	9	19	12	15.20	12	10	10	10	11	12	13	13	13
10	1,187	9	19	12	13.70	12	13	13	13	14	12	13	13	13
11	996	8	17	13	10.90	14	11	11	11	11	9	9	8	8
12	996	8	17	13	10.90	12	9	10	9	10	9	8	8	8
13	1,992	11	24	17	12.90	16	12	12	12	13	12	12	11	11
14	3,179	14	30	20	17.10	17	17	17	17	18	16	17	17	17
15	403	5	11	9	7.60	9	6	6	6	6	6	5	5	5
16	403	5	11	9	7.60	7	6	6	6	6	6	5	5	5
17	805	7	15	12	8.40	9	8	8	8	8	7	7	7	7
18	3,984	16	34	22	18.80	17	26	26	26	28	24	24	24	26
Root: 19	3,984	16	34	22	25.20	17	26	26	26	28	24	24	24	26

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[*] Numbers in shaded cells indicate duct sizes at the lower bound.

TABLE A-2
Duct Sizes of the Large Office Building in Oklahoma (8760-Hour Schedule, Aluminum Ducts)
Unit: in. (1 in. = 0.0254 m)

Duct Section	Max. Airflow, cfm	Min. Size	Max. Size	Equal Friction	Static Regain	T-method				VAV Optimization Procedure			
						TSAL E Rate	Okla. E Rate	Minn. E Rate	N.Y. E Rate	TSAL E Rate	Okla. E Rate	Minn. E Rate	N.Y. E Rate
1	8,678	24	50	30	24	26	26	26	26	24	24	24	26
2	2,699	13	28	18	14	13	13	13	14	13	13	13	13
3	1,800	11	22	16	14	12	12	12	12	11	11	11	11
4	1,350	9	20	14	14	9	9	9	10	9	9	9	11
5	900	8	16	12	12	8	8	8	9	8	8	8	9
6	450	6	11	10	9	7	7	7	7	6	7	7	7
7	900	8	16	12	9	8	8	8	8	8	8	8	8
8	1,637	10	22	16	11	11	11	11	11	10	10	10	10
9	910	8	16	12	9	9	9	9	9	8	8	8	8
10	728	7	15	11	9	7	8	8	8	8	7	7	7
11	546	6	13	10	9	6	6	6	6	6	6	6	6
12	364	5	10	9	9	5	5	5	6	6	5	5	6
13	182	4	7	7	7	4	4	4	4	4	4	4	5
14	182	4	7	7	5	4	4	4	4	4	4	4	4
15	728	7	15	11	9	7	8	8	8	7	7	7	7
16	182	4	7	7	5	4	4	4	4	4	4	4	4
17	546	6	13	10	9	6	6	6	6	6	6	6	6
18	364	5	10	9	9	5	5	5	5	5	5	5	5
19	182	4	7	7	7	4	4	4	4	4	4	4	4
20	4,341	16	36	22	18	19	19	19	20	16	20	17	20
21	728	7	15	11	9	7	7	7	8	7	7	7	7
22	546	6	13	10	9	6	6	6	6	7	6	6	7
23	364	5	10	9	9	5	5	5	6	5	5	6	5
24	182	4	7	7	7	4	4	4	4	4	4	5	4
25	3,614	15	32	22	18	15	16	16	16	15	16	16	16
26	723	7	14	11	9	7	7	7	7	7	7	7	7
27	1,445	10	20	15	14	10	10	10	11	10	10	10	10
28	1,084	8	18	13	14	9	9	9	9	8	8	8	8
29	723	7	14	11	12	8	8	8	8	7	7	7	7
30	361	5	10	9	9	6	6	6	6	5	5	5	5
31	1,445	10	20	15	14	12	12	12	13	11	11	10	10
32	1,084	8	18	13	14	10	10	10	10	8	8	9	9
33	723	7	14	11	12	8	9	9	9	7	8	7	7
34	361	5	10	9	9	7	7	7	7	6	6	6	6

TABLE A-3
Duct Sizes of the Large Office Building in Minnesota (8760-Hour Schedule, Aluminum Ducts)
Unit: in. (1 in. = 0.0254 m)

Duct Section	Max. Airflow, cfm	Min. Duct Size	Max. Duct Size	Equal Friction	ASHRAE Ans.	Static Regain	T-method				VAV Optimization Procedure		
						TSAL E Rate	Okla. E Rate	Minn. E Rate	N.Y. E Rate	TSAL E Rate	Okla. E Rate	Minn. E Rate	N.Y. E Rate
1	7,584	22	48	28	22	24	24	24	26	22	24	22	24
2	2,481	12	26	18	14	13	13	13	13	12	12	12	12
3	1,654	10	22	15	14	11	11	11	12	10	11	11	11
4	1,240	9	19	14	14	9	9	9	9	9	9	10	11
5	827	7	16	12	12	8	8	8	8	7	8	10	8
6	413	5	11	9	9	6	6	6	7	5	6	6	7
7	827	7	16	12	9	6	6	6	6	7	7	7	7
8	1,254	9	19	14	9	10	10	10	10	9	9	9	9
9	697	7	14	11	9	8	8	8	8	7	8	7	7
10	558	6	13	10	9	7	7	7	7	7	6	6	7
11	418	5	11	9	9	6	6	6	6	5	5	6	6
12	279	5	9	8	9	5	5	5	5	5	5	5	5
13	139	3	6	6	6	4	4	4	4	4	4	4	4
14	139	3	6	6	5	3	3	3	3	3	3	3	3
15	558	6	13	10	7	7	7	7	7	6	6	6	6
16	139	3	6	6	4	3	3	3	3	3	3	3	3
17	418	5	11	9	7	6	6	6	6	5	5	5	5
18	279	5	9	8	7	5	5	5	5	5	5	5	5
19	139	3	6	6	5	4	4	4	4	3	4	3	4
20	3,849	15	34	22	18	18	18	18	19	16	17	16	19
21	558	6	13	10	9	7	7	7	7	6	6	6	6
22	418	5	11	9	9	6	6	6	6	5	5	6	6
23	279	5	9	8	9	5	5	5	5	5	5	6	5
24	139	3	6	6	6	4	4	4	4	5	4	4	4
25	3,291	14	30	20	18	15	15	15	16	14	14	16	17
26	658	7	14	11	9	7	7	7	7	7	7	7	7
27	1,317	9	20	14	14	10	10	10	10	9	9	9	9
28	987	8	17	13	14	8	8	8	9	8	8	9	8
29	658	7	14	11	12	7	7	7	8	7	7	7	7
30	329	5	10	9	9	6	6	6	6	5	6	5	5
31	1,317	9	20	14	14	12	12	12	12	9	11	9	9
32	987	8	17	13	14	9	9	9	10	8	9	9	9
33	658	7	14	11	12	8	8	8	9	7	8	7	8
34	329	6	10	9	9	7	7	7	7	6	6	6	6

TABLE A-4
Duct Sizes of the Large Office Building in Oklahoma (8760-Hour Schedule, Ga. Steel Ducts)
Unit: in. (1 in. = 0.0254 m)

Duct Section	Max. Airflow, cfm	Min. Size, in.	Max. Size, in.	Equal Friction	Static Regain	T-method			VAV Opt. Procedure			
						Okla. E Rate	Minn. E Rate	N.Y. E Rate	Okla. E Rate	Minn. E Rate	N.Y. E Rate	Okla. E Rate (Doubled Duct Cost)
1	8,678	24	50	30	24	26	26	26	24	24	26	24
2	2,699	13	28	18	14	14	14	14	13	13	13	13
3	1,800	11	22	16	14	12	12	12	11	11	12	11
4	1,350	9	20	14	14	9	9	9	9	10	9	9
5	900	8	16	12	12	8	8	8	8	8	8	9
6	450	6	11	10	9	7	6	7	7	6	7	6
7	900	8	16	12	9	8	8	8	8	8	8	8
8	1,637	10	22	16	11	11	11	11	10	10	10	10
9	910	8	16	12	9	9	9	9	8	8	8	8
10	728	7	15	11	9	7	7	7	7	7	8	7
11	546	6	13	10	9	6	6	6	6	6	6	6
12	364	5	10	9	9	5	5	5	5	5	5	5
13	182	4	7	7	7	4	4	4	4	4	4	4
14	182	4	7	7	5	4	4	4	4	4	4	4
15	728	7	15	11	9	7	7	8	7	7	7	7
16	182	4	7	7	5	4	4	4	4	4	4	4
17	546	6	13	10	9	6	6	6	7	6	6	6
18	364	5	10	9	9	5	5	5	5	6	5	5
19	182	4	7	7	7	4	4	4	4	4	4	4
20	4,341	16	36	22	18	17	17	17	16	17	18	16
21	728	7	15	11	9	7	7	7	7	7	7	7
22	546	6	13	10	9	6	6	6	7	7	6	7
23	364	5	10	9	9	5	5	5	5	6	5	5
24	182	4	7	7	7	4	4	4	4	4	5	4
25	3,614	15	32	22	18	15	15	16	16	16	18	15
26	723	7	14	11	9	7	7	7	7	7	7	7
27	1,445	10	20	15	14	10	10	10	10	10	10	10
28	1,084	8	18	13	14	8	8	9	8	9	8	8
29	723	7	14	11	12	7	7	8	7	7	7	8
30	361	5	10	9	9	6	6	6	5	5	5	5
31	1,445	10	20	15	14	12	12	12	10	10	10	10
32	1,084	8	18	13	14	9	9	10	9	9	9	9
33	723	7	14	11	12	8	8	9	7	7	8	7
34	361	5	10	9	9	7	7	7	6	6	6	5

TABLE A-5
Duct Sizes of the Large Office Building in Oklahoma with Setback Control Operation (Optimized with Aluminum Ducts and Okla. Electric Rate)
Unit: in. (1 in. = 0.0254 m)

Duct Section	Max. Airflow, cfm	Min. Size	Max. Size	Setback-control Schedule			8760-hour Schedule Vav Opt.
				Equal Friction	T-method	Vav Opt	
1	8789	24	50	30	26	26	24
2	2730	13	28	19	14	13	13
3	1820	11	22	16	12	11	11
4	1365	9	20	15	10	9	9
5	910	8	16	13	8	8	8
6	455	6	11	9	7	7	7
7	910	8	16	13	8	8	8
8	1639	10	22	16	11	11	10
9	911	8	16	13	9	9	8
10	729	7	15	11	8	7	7
11	546	6	13	10	6	6	6
12	364	5	10	9	6	5	5
13	182	4	7	7	4	4	4
14	182	4	7	7	4	4	4
15	729	7	15	11	8	7	7
16	182	4	7	7	4	4	4
17	546	6	13	10	6	7	6
18	364	5	10	9	5	5	5
19	182	4	7	7	4	4	4
20	4420	16	36	22	20	18	20
21	729	7	15	11	8	7	7
22	546	6	13	10	6	6	6
23	364	5	10	9	5	6	5
24	182	4	7	7	4	4	4
25	3691	15	32	22	16	16	16
26	738	7	14	11	7	7	7
27	1477	10	20	15	10	10	10
28	1107	8	18	13	9	8	8
29	738	7	14	11	8	7	7
30	369	5	10	9	6	6	5
31	1477	10	20	15	13	11	11
32	1107	8	18	13	10	9	8
33	738	7	14	11	9	8	8
34	369	5	10	9	7	7	6

TABLE A-6
Duct Sizes of the Large Office Building in Minnesota with Setback Control Operations (Optimized with Aluminum Ducts and Minn. Electric Rate)
Unit: in. (1 in. = 0.0254 m)

Duct Section	Max. Airflow, cfm	Min. Size	Max. Size	Setback-control Schedule			8760-hour Schedule VAV Opt.
				Equal Friction	T-method	VAV Opt.	
1	7,574	22	48	28	24	22	22
2	2,498	12	26	18	13	13	12
3	1,665	10	22	16	11	11	11
4	1,249	9	19	14	9	9	10
5	833	7	16	12	8	8	10
6	416	5	11	9	6	6	6
7	833	7	16	12	7	7	7
8	1,213	9	19	14	10	9	9
9	674	7	14	11	8	7	7
10	539	6	13	10	7	6	6
11	404	5	11	9	6	6	6
12	269	4	9	8	5	5	5
13	135	3	6	6	4	4	4
14	135	3	6	6	3	3	3
15	539	6	13	10	7	6	6
16	135	3	6	6	3	3	3
17	404	5	11	9	6	6	5
18	269	4	9	8	5	4	5
19	135	3	6	6	4	4	3
20	3,864	15	34	22	18	16	16
21	539	6	13	10	7	6	6
22	404	5	11	9	6	6	6
23	269	4	9	8	5	5	6
24	135	3	6	6	4	5	4
25	3,325	14	30	20	15	16	16
26	665	7	14	11	7	7	7
27	1,330	9	20	14	10	9	9
28	997	8	17	13	8	8	9
29	665	7	14	11	7	7	7
30	332	5	10	9	6	6	5
31	1,330	9	20	14	12	10	9
32	997	8	17	13	9	9	9
33	665	7	14	11	8	8	7
34	332	6	10	9	7	6	6

TABLE A-7
Cost Comparison Between Different Duct Design Methods (Aluminum Ducts)
Unit:$

Duct System	Duct Design Method	TSAL Electric Rate					Okla. Electric Rate				
		Duct Cost	Fan Cost	Opr. Cost	L.C.C	Saving% of VAV Opt.	Duct Cost	Fan Cost	Opr. Cost	L.C.C	Saving% of VAV Opt.
ASHRAE example	Equal friction	9,131	2,110	2,022	13,263	14.4	9,131	2,110	6,051	17,293	9.4
	ASHRAE ans.	7,914	2,110	2,101	12,125	6.4	7,914	2,110	6,207	16,231	3.5
	Static regain	8,022	2,110	2,138	12,270	7.5	8,022	2,110	6,276	16,407	4.5
	T-method	7,331	2,110	2,224	11,665	2.7	7,539	2,110	6,311	15,960	1.8
	VAV opt.	6,995	2,110	2,243	11,348	0.0	7,107	2,110	6,451	15,668	0.0
Building in Okla.	Equal friction	8,945	2,575	2,950	14,470	17.3	8,945	2,575	7,702	19,223	12.2
	Static regain	7,625	2,575	3,104	13,303	10.0	7,625	2,575	7,965	18,165	7.0
	T-method	6,535	2,575	3,116	12,227	2.1	6,614	2,575	7,904	17,093	1.2
	VAV opt.	6,214	2,575	3,178	11,967	0.0	6,372	2,575	7,940	16,887	0.0
Building in Minn.	Equal friction	8,440	2,420	2,559	13,419	18.7	8,440	2,420	6,917	17,777	12.2
	Static regain	7,509	2,420	2,642	12,571	13.3	7,509	2,420	7,060	16,989	8.1
	T-method	6,288	2,420	2,662	11,370	4.1	6,288	2,420	7,098	15,806	1.2
	VAV opt.	5,667	2,420	2,817	10,905	0.0	6,015	2,420	7,182	15,617	0.0

TABLE A-8
Cost Comparison Between Different Duct Design Methods (Aluminum Ducts)
Unit: $

Duct System	Duct Design Method	Minn. Electric Rate					N.Y. Electric Rate				
		Duct Cost	Fan Cost	Opr. Cost	L.C.C	Saving% of VAV Opt.	Duct Cost	Fan Cost	Opr. Cost	L.C.C	Saving% of VAV Opt.
ASHRAE example	Equal friction	9,131	2,110	6,423	17,664	9.2	9,131	2,110	10,182	21,424	6.1
	ASHRAE ans.	7,914	2,110	6,616	16,640	3.6	7,914	2,110	10,535	20,560	2.2
	Static regain	8,022	2,110	6,693	16,408	2.2	8,022	2,110	10,705	20,837	3.5
	T-method	7,524	2,110	6,742	16,377	2.0	7,913	2,110	10,524	20,547	2.1
	VAV opt.	7,128	2,110	6,806	16,044	0.0	7,459	2,110	10,546	20,115	0.0
Building in Okla.	Equal friction	8,945	2,575	8,490	20,010	11.5	8,945	2,575	14,197	25,718	7.8
	Static regain	7,625	2,575	8,825	19,025	6.9	7,625	2,575	14,861	25,060	5.3
	T-method	6,614	2,575	8,783	17,972	1.5	6,835	2,575	14,631	24,042	1.3
	VAV opt.	6,236	2,575	8,897	17,707	0.0	6,478	2,575	14,668	23,720	0.0
Building in Minn.	Equal friction	8,440	2,420	7,676	18,536	11.7	8,440	220	12,385	23,246	8.7
	Static regain	7,509	2,420	7,861	17,790	8.0	7,509	2,420	12,745	22,673	6.4
	T-method	6,288	2,420	7,901	16,609	1.4	9,131	2,110	10,182	21,424	6.1
	VAV opt.	5,862	2,420	8,087	16,369	0.0	7,914	2,110	10,535	20,560	2.2

TABLE A-9
Cost Comparison Between Different Duct Design Methods When the Large Office Building in Oklahoma Uses Galvanized Steel Ducts

Electric Rate	Duct Design Method	Duct Cost (Ga. Steel)	Fan Cost	Operating Cost	L.C. Cost	Saving% of VAV Opt.
Okla. electric rate	Equal friction	11,474	2,575	7,845	21,893	12.7
	Static regain	9,915	2,575	8,223	20,712	7.7
	T-method	8,362	2,575	8,338	19,276	0.8
	VAV opt.	7,957	2,575	8,591	19,123	0.0
Minn. electric rate	Equal friction	11,474	2,575	8,671	22,720	11.0
	Static regain	9,915	2,575	9,151	21,641	6.6
	T-method	8,349	2,575	9,367	20,292	0.4
	VAV opt.	8,025	2,575	9,613	20,213	0.0
N.Y. electric rate	Equal friction	11,474	2,575	14,557	28,606	6.8
	Static regain	9,915	2,575	15,506	27,996	4.8
	T-method	8,491	2,575	15,765	26,831	0.7
	VAV opt.	8,329	2,575	15,750	26,654	0.0

TABLE A-10
Cost Comparison Between Different Duct Design Methods When Two Different System Operating Schedules Are Used

Duct System	Duct Design Method		Duct Cost (Aluminum)	Fan Cost	Opr. Cost	L.C. Cost	Saving% of VAV Opt.
LOB in Okla. with Okla. electric rate	Setback control	Equal friction	9,029	2,575	5,649	17,254	13.6
		T-method	6,788	2,575	5,802	15,165	1.7
		VAV opt.	6,488	2,575	5,845	14,908	0.0
	8760-hr	VAV opt.	6,372	2,575	7,940	16,887	11.7
LOB in Minn. with Minn. electric rate	Setback control	Equal friction	8,467	2,420	6,885	17,771	12.5
		T-method	6,288	2,420	7,110	15,818	1.7
		VAV opt.	5,951	2,420	7,182	15,554	0.0
	8760-hr	VAV opt.	5,862	2,420	8,087	16,369	5.0

Kim, T., J.D. Spitler, R.D. Delahoussaye. 2002. Optimum Duct Design for Variable Air Volume Systems, Part 2: Optimization of VAV Duct Systems. *ASHRAE Transactions*. 108(1):105-127.