Ground Heat Exchangers Introduction to Modeling

Part 1

Jeffrey D. Spitler
spitler@okstate.edu
https://hvac.okstate.edu/

OKLAHOMA STATE UNIVERSITY

GSHP

Why?

- Numerical models of ground heat exchangers (GHE) are needed for:
  - Design purposes using simulation-based design tools (GLHEPRO, EED)
  - Energy analysis in whole building energy simulation tools (e.g. EnergyPlus)
  - Optimizing thermal design, hydraulic design, system configuration, controls and economics.

Goals

- The main goals of this lecture are to introduce:
  - The problem
  - Concepts used for modeling GHE
  - A very simple approach that is not state-of-the-art, but which will give the student a better understanding of GHE modeling.
- This lecture is not intended to:
  - Give a detailed survey of the entire field.
  - Introduce the latest methods being applied.

Introduction to ground heat exchanger modeling

- 3-dimensional conduction heat transfer problem:
  - Very complex geometry.
  - Wide range of length scales. (mm to 100s of m)
  - Wide range of time scales. (minutes to decades)
  - Generally impractical to solve numerically (FEM/FDM/FVM)
    - Yes, it can be done for a small number of boreholes for research purposes for short time periods.
    - But not for large numbers of boreholes
  - A wide range of simplified methods have been developed. Some better than others. They work until they don’t.

GHE modeling

- Analytical
  - Line source
  - Cylinder source
- Numerical
  - Finite difference, finite volume, finite element
  - Custom thermal networks
- Response Factors
  - Claesson, Eskilson, Hellström – “long time step”
  - Yavuzturk and Spitler(1999) – “short time step”
  - Xu and Spitler (2006) – “short time step”
- Hybrid: Response Factors and Numerical
  - Xu and Spitler (2006)

Analytical models

- Infinite Line Source
  - Represents entire heat rejection/extraction as a line source in infinite media; i.e. a one-dimensional radial only problem.
  - Used to compute temperature of the ground at the borehole wall.
- Infinite Cylinder Source
  - Several variations which differ based on what's inside the cylinder.
  - Also used to compute temperature of the ground at the borehole wall.
  - See Carslaw and Jaeger. 1947. Conduction of Heat in Solids or Thomas Ray Young's MS thesis (2004)
  - Young's model was considered for g-function generation in GLHEPRO, but less-than-desirable accuracy at times under 6 hours led us to replace it with the Xu and Spitler (2006) numerical model.

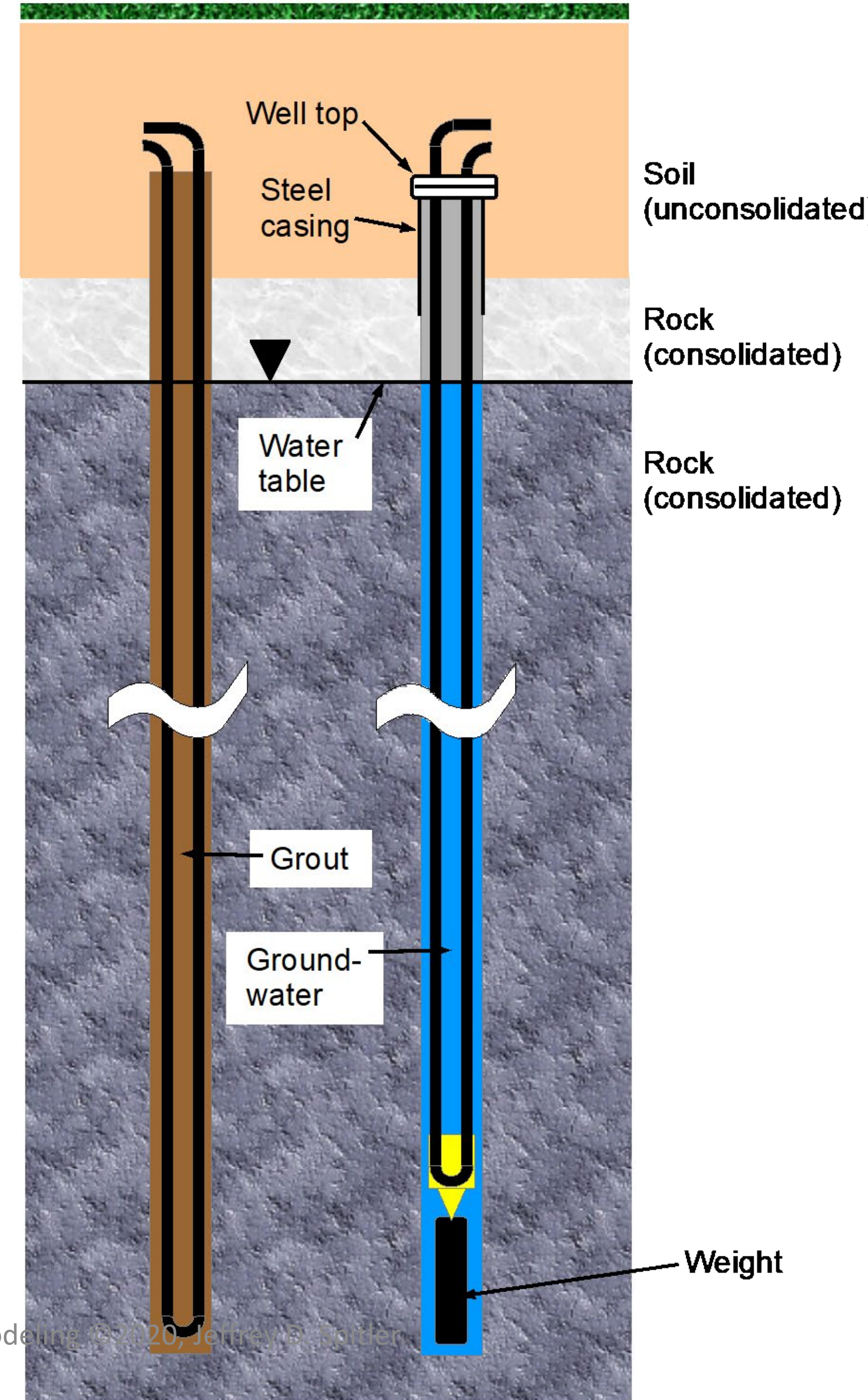
Analytical models

- Finite line source (FLS)
  - Represents heat rejection/extraction as a finite line source in infinite media; end effects can be included.
  - Entire borehole can be represented with a single FLS
  - Or, borehole can be represented with multiple FLS, each of which has its own heat input.

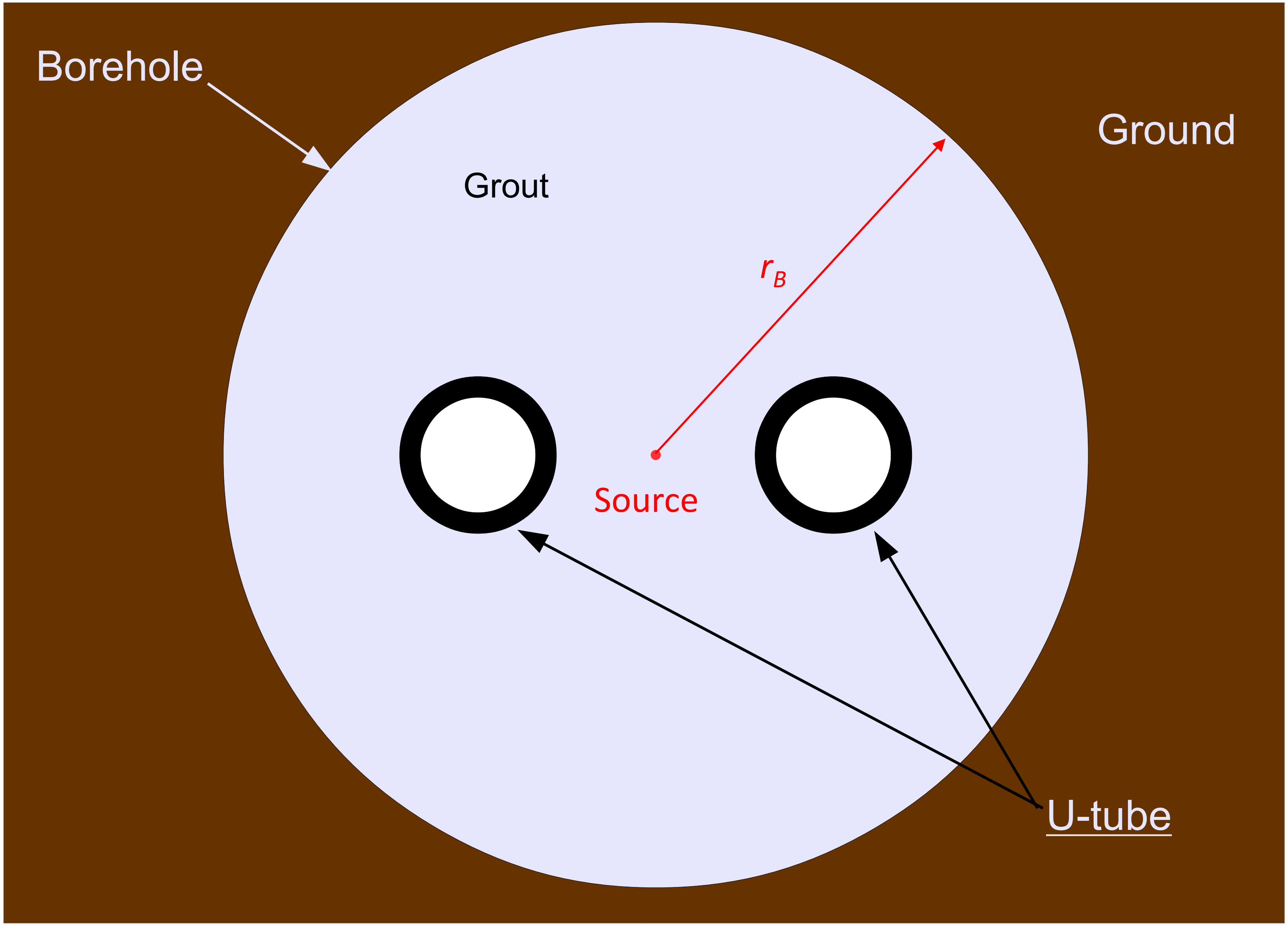
Line source

- Can also be thought of as a point source in a planar sheet.
- Erroneously said to be developed by Lord Kelvin.
- Not in the commonly cited 1880 paper, but can be derived from point source in the paper. (See Thomson 1884)
- But the point source solution comes from Fourier.

North American and Scandinavian Borehole designs: Grouted (left) Groundwater-filled (right)



Temperature calculated at r_B



Line source

- Temperature at borehole wall calculated assuming line source at center of borehole and borehole is filled with "ground"
- Ingersoll and Plass (1948)

$$\Delta T = \frac{q}{4\pi k_{soil}} \int_{x}^{\infty} \frac{e^{-\beta}}{\beta} d\beta$$

$$x = \frac{r^2}{4\alpha_{soil} t}$$

Increase in temperature at radius *r* of infinite medium exposed to line source of strength *q*

Line source

ΔT = change in ground temperature at a distance r from the line source (°C) or (°F)

q = heat transfer rate per length of line source $\left(\frac{W}{m}\right)$ or $\left(\frac{Btu}{h \cdot ft}\right)$

t = time duration of heat input q (s)

r = radius from the line source (m) or (ft)

α_{soil} = soil thermal diffusivity $\left(\frac{m^2}{s}\right)$ or $\left(\frac{ft^2}{s}\right)$

k_{soil} = conductivity of the soil $\left(\frac{W}{mK}\right)$ or $\left(\frac{Btu}{h \cdot ft \cdot {}^\circ F}\right)$

Ground Heat Exchangers Introduction to Modeling

Part 2

Jeffrey D. Spitler
spitler@okstate.edu
https://hvac.okstate.edu/

OKLAHOMA STATE UNIVERSITY

GSHP U

Line source

$$\int_x^{\infty} \frac{e^{-\beta}}{\beta} d\beta$$

Is the exponential integral, sometimes called E_1 [*cf*. Hellström 1991 Gautschi and Cahill 1964], or I [*cf.* Young(2004) or Ingersoll and Plass(1948)]

$E_1(x)$ or $I(x)$ is tabulated in several references such as Abramowitz and Stegun, who also give functional approximations. See Gautschi and Cahill (1964)

Young (pp. 12-14) gives two approximations that cover different ranges.

Line Source

A simpler approximation:

$$\int_{r^2/4\alpha t}^{\infty} \frac{e^{-\beta}}{\beta} d\beta = E_1\left(\frac{r^2}{4\alpha t}\right) \approx \ln\left(\frac{4\alpha t}{r^2}\right) - \gamma$$

$$\gamma = 0.57722\ldots \quad \text{(Euler's constant)}$$

Is claimed to have a maximum error of 2% when $\alpha t/r^2 \geq 5$

For "average rock" with thermal diffusivity of $7.4 \cdot 10^{-7} \frac{m^2}{s}$ and a borehole diameter of 140 mm, t at the limit $\alpha t/r^2 = 5$ is 9.2 hours

Line Source

An improved approximation:

$$\int_{r^2/4\alpha t}^{\infty} \frac{e^{-\beta}}{\beta} d\beta = E_1\left(\frac{r^2}{4\alpha t}\right) \approx \ln\left(\frac{4\alpha t}{r^2}\right) - \gamma - \frac{1}{4}\left[\frac{r^2}{\alpha t} - \left(\frac{r^2}{4\alpha t}\right)^2\right]$$

$$\gamma = 0.57722... \quad \text{(Euler's constant)}$$

Has a maximum error of 1% when $\alpha t/r^2 \geq 0.5$. (But can have enormous errors at small values of $\alpha t/r^2$!)

For "average rock" with thermal diffusivity of $7.4 \cdot 10^{-7} \frac{m^2}{s}$ and a borehole diameter of 140 mm, t at the limit $\alpha t/r^2 = 0.5$ is 0.92 hours

Line source

- So, for now, we will just use:

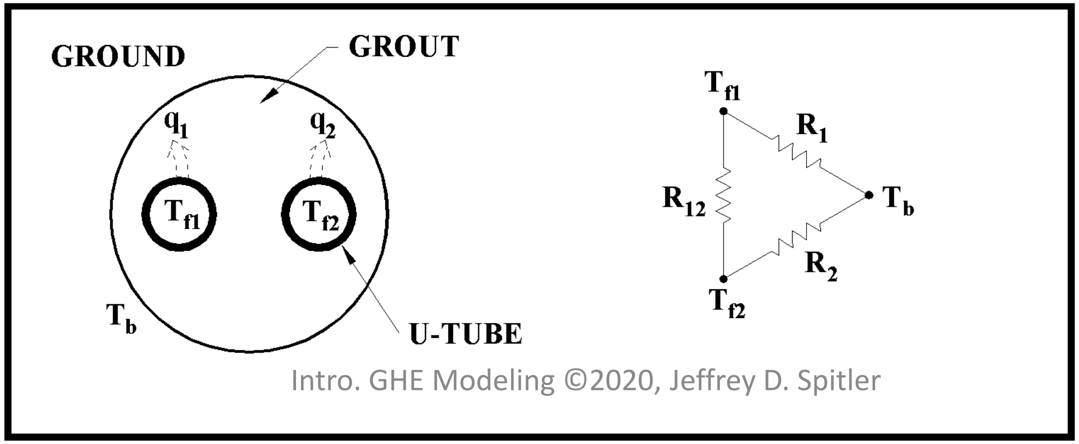
$$\int_{r^2/4\alpha t}^{\infty} \frac{e^{-\beta}}{\beta} d\beta = E_1\left(\frac{r^2}{4\alpha t}\right) \approx \ln\left(\frac{4\alpha t}{r^2}\right) - \gamma$$

$$\gamma = 0.57722\ldots \quad \text{(Euler's constant)}$$

- Possible separate discussion on history and application of line source.
- Warning: several of the references, e.g. Ingersoll and Plass (1948) and Carslaw and Jaeger (1959) have typos. The above expressions are correct.

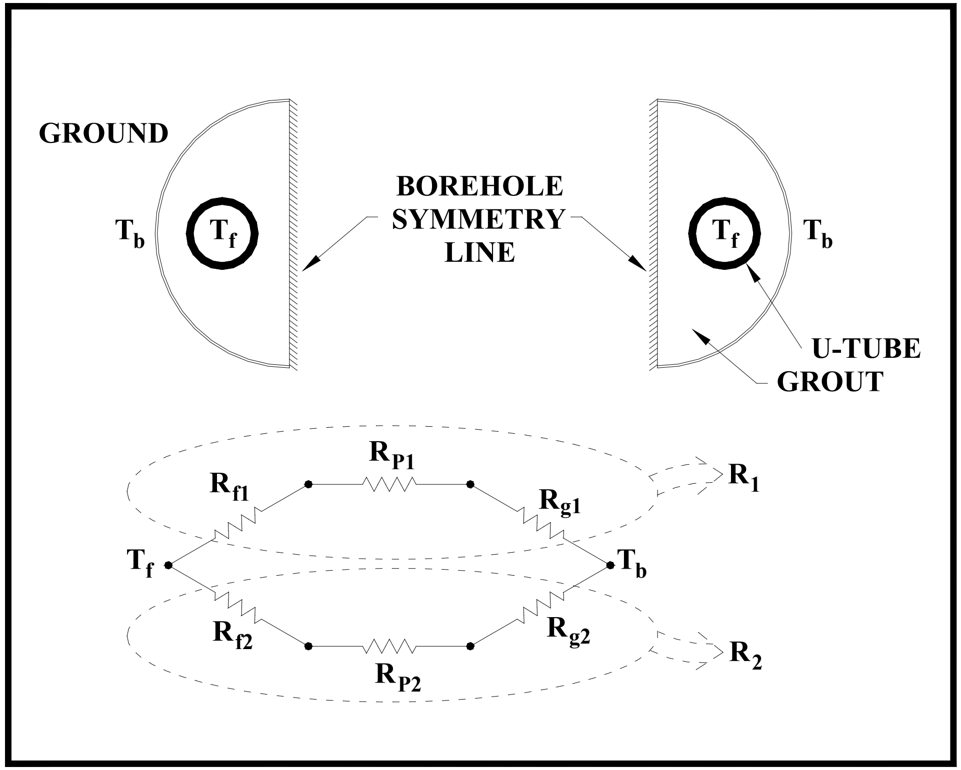
Line Source

- So far, this would give us the temperature at the borehole wall with a constant heat input.
- [And this is an approximation because the borehole is not filled with ground.]
- To get average fluid temperature in the borehole, the borehole resistance is typically used.



From Young (2004) and Hellström (1991)

Borehole Resistance



From Young (2004)

Neglecting short-circuiting resistance,

$$R_{total} = R_{grout} + \frac{R_{pipe} + R_{fluid}}{2}$$

Borehole Resistance

- R_{pipe}, R_{fluid} (convective resistance) readily computed
- R_{grout}, the resistance between the outer pipe surfaces and the borehole wall is more difficult. It can be computed with:
  - Numerical procedure, like Yavuzturk (1999)
  - Multipole method (Claesson and Hellström 2011)
  - Shape factor method (Paul 1996)
  - See Javed and Spitler (2017) for analysis of the accuracy of different methods.
  - Recent developments suggest lower-order closed-form multipole approach is probably "best." See Claesson and Javed (2018, 2019)

Calculation procedures

- Numerical procedures work, but have difficulty with:

- Multipole method is very accurate; may have limitations with boreholes that are cased.
- Shape factor method has issues with assumptions used (uniform flux at pipe wall rather than uniform temperature inside) that limit accuracy.

Short-circuiting

- Short circuiting increases effective borehole thermal resistance.
- Hellström denotes this as R_b*, the effective borehole thermal resistance.
- $R_b* > R_b$
- For typical North American boreholes of depth 80 m, R_b* is only a little larger than R_b.
- But as boreholes get deeper and/or flow rates get lower, the difference widens.

Short-circuiting (An aside)

- Fair amount of confusion over this issue and a related issue: how to determine the mean fluid temperature in the borehole.
- Some authors have rightly pointed out the mean fluid temperature is NOT the simple mean of the EFT and ExFT from the borehole. (True.)
- They have developed a range of procedures for estimating the actual mean fluid temperature. (Fine).
- Unfortunately, many of these authors did not understand that R_b* treats this problem.

Short-circuiting (An aside)

- Specifically, you may use:
  - R_b* with the simple mean fluid temperature.
  - R_b with a better-computed mean fluid temperature.
  - Or, for relatively shallow (< 100m) boreholes, R_b with the simple mean fluid temperature. The difference will only be a few %.
- Expressions for R_b*
  - See: Claesson and Hellström (2011).
  - Two limiting cases: uniform borehole wall temperature and uniform heat flux are given.
  - Mean value of the two limiting cases may be used.

Ground Heat Exchangers Introduction to Modeling

Part 3

Jeffrey D. Spitler
spitler@okstate.edu
https://hvac.okstate.edu/

Expression for mean fluid temperature

For a constant heat rejection rate, the mean fluid temperature may be expressed as:

$$T(t) = \frac{q}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil} \cdot t}\right) + q \cdot R_b^* + T_{ff}$$

See next slide for nomenclature

GSHP U

$$T(t) = \frac{q}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil} \cdot t}\right) + q \cdot R_b^* + T_{ff}$$

T = Borehole fluid temperature (°C) or (°F) — Mean fluid temperature

t = time duration of heat input (s)

T_{ff} = far field temperature of the soil (°C) or (°F)

q = heat transfer rate per length of line source $\left(\frac{W}{m}\right)$ or $\left(\frac{Btu}{h \cdot ft}\right)$ — This is positive when injecting heat into the ground, i.e. when the heat pump is cooling.

r_{bh} = radius of the borehole (m) or (ft)

R_b^* = steady state borehole resistance $\left(\frac{mK}{W}\right)$ or $\left(\frac{h \cdot ft \cdot {}^\circ F}{Btu}\right)$ — Effective borehole thermal resistance

α_{soil} = soil thermal diffusivity $\left(\frac{m^2}{s}\right)$ or $\left(\frac{ft^2}{s}\right)$

k_{soil} = conductivity of the soil $\left(\frac{W}{mK}\right)$ or $\left(\frac{Btu}{h \cdot ft \cdot {}^\circ F}\right)$

Computing heat pump EFT

- Usually of interest to compute heat pump EFT (equivalent to the borehole ExFT).
- The simplest approach, usually suitable for time steps of 15 minutes or more is to assume the borehole and fluid are in quasi-steady conditions and that the mean fluid temperature is the simple mean fluid temperature. Then:

$$qL = \dot{m}c_p\Delta T$$

$$\Delta T = \frac{qL}{\dot{m}c_p}$$ This is ΔT across borehole; L is active length of borehole.

$$\frac{\Delta T}{2} = \frac{qL}{2\dot{m}c_p}$$ Temperature difference between mean fluid temperature and borehole ExFT is $\Delta T/2$

Heat pump EFT

The fluid exiting the borehole and entering the heat pump then has a temperature expressed as:

$$T_{ExBH}(t) = \frac{q}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil} \cdot t}\right) + q \cdot R_b^* + T_{ff} - \frac{qL}{2\dot{m}c_p}$$

This expression could be used to calculate heat pump EFT for a fixed heat transfer rate. Except for analyzing thermal response tests… not so useful.

q varies with time, too!

- Requires deconvolution of actual load profile into step functions;
- Computation of response for each step function
- Convolution of individual responses (i.e. superposition of the individual temperature responses)

Temporal superposition example

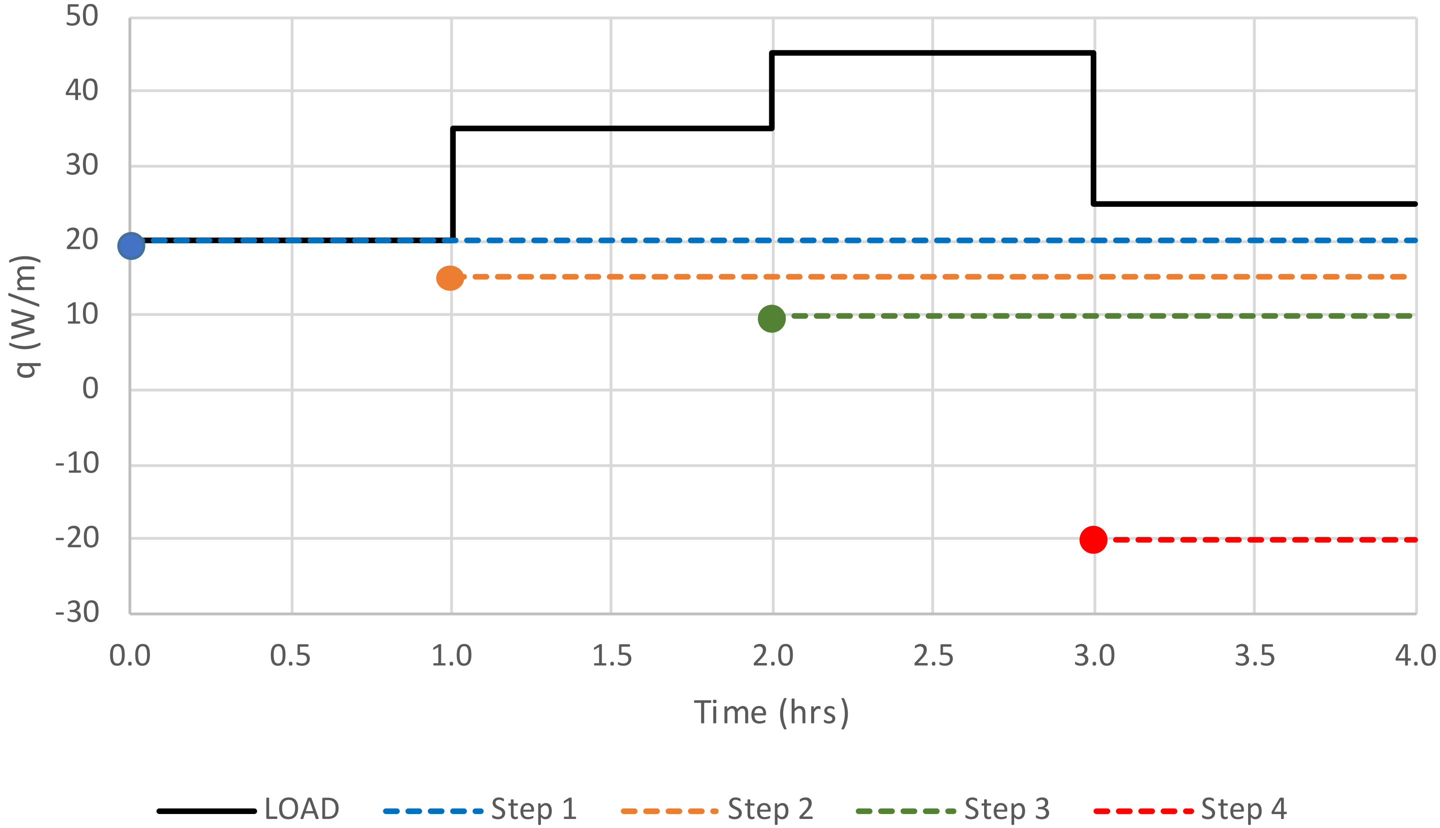


Figure originally drawn for Spitler and Bernier (2016) and is further discussed there.

Temporal superposition example

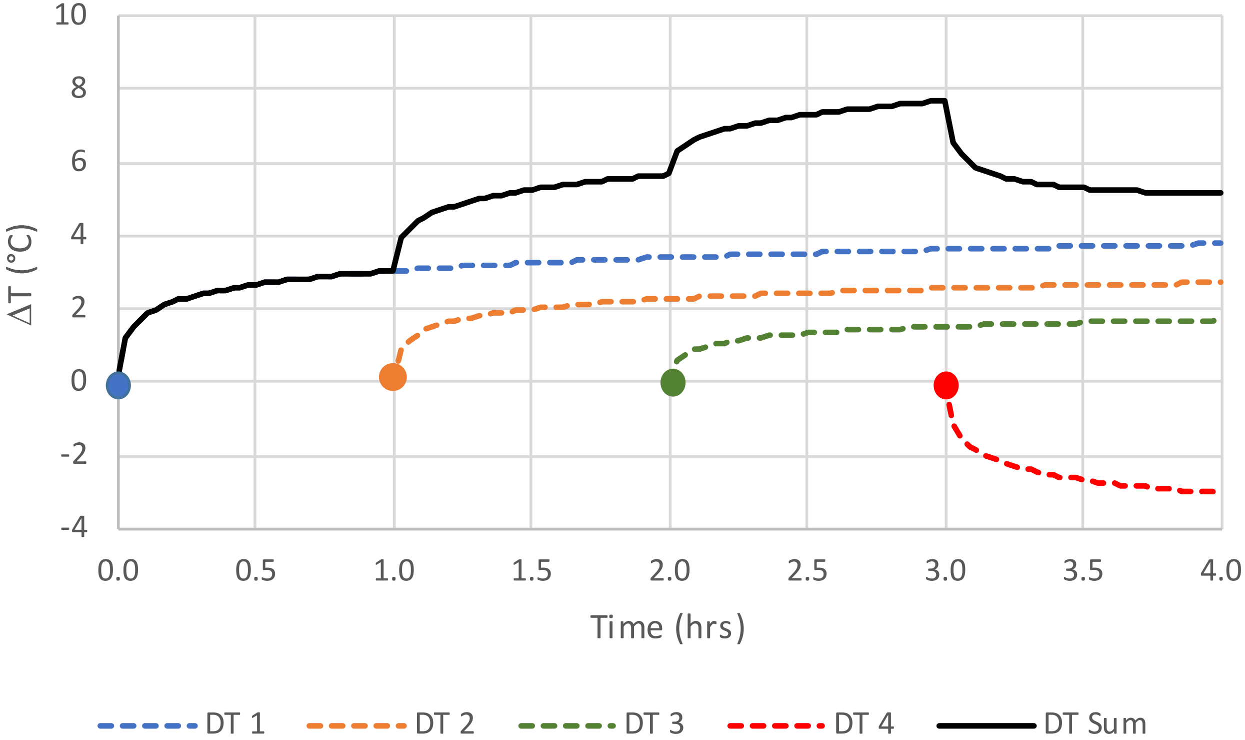


Figure originally drawn for Spitler and Bernier (2016) and is further discussed there.

Use superposition to determine borehole wall temperature

Using fixed time intervals of time Δt with different heat rejection rates: $q_1, q_2, q_3, etc.$

For the 1st time interval

$$\Delta T_1 = \frac{q_1}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil} \cdot \Delta t}\right)$$

For the 2nd time interval

$$\Delta T_2 = \frac{q_1}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil} \cdot 2\Delta t}\right) + \frac{(q_2 - q_1)}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil} \cdot \Delta t}\right)$$

For the 3rd time interval

$$\Delta T_3 = \frac{q_1}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil}\cdot 3\Delta t}\right) + \frac{(q_2 - q_1)}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil}\cdot 2\Delta t}\right) + \frac{(q_3 - q_2)}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil}\cdot 1\Delta t}\right)$$

For the nth time interval

$$\Delta T_n = \sum_{i=1}^{n} \frac{(q_i - q_{i-1})}{4\pi k_{soil}} \cdot E_1\left(\frac{r_{bh}^2}{4\alpha_{soil} \cdot (n - i + 1)\Delta t}\right)$$

$$T_{ExBH}(t) = \Delta T_n + q \cdot R_b^* + T_{ff} - \frac{qL}{2\dot{m}c_{p,fluid}}$$

$$T_{MF}(t) = \Delta T_n + q \cdot R_b^* + T_{ff}$$ Mean fluid temperature

Ground Heat Exchangers Introduction to Modeling

Part 4

Jeffrey D. Spitler
spitler@okstate.edu
https://hvac.okstate.edu/

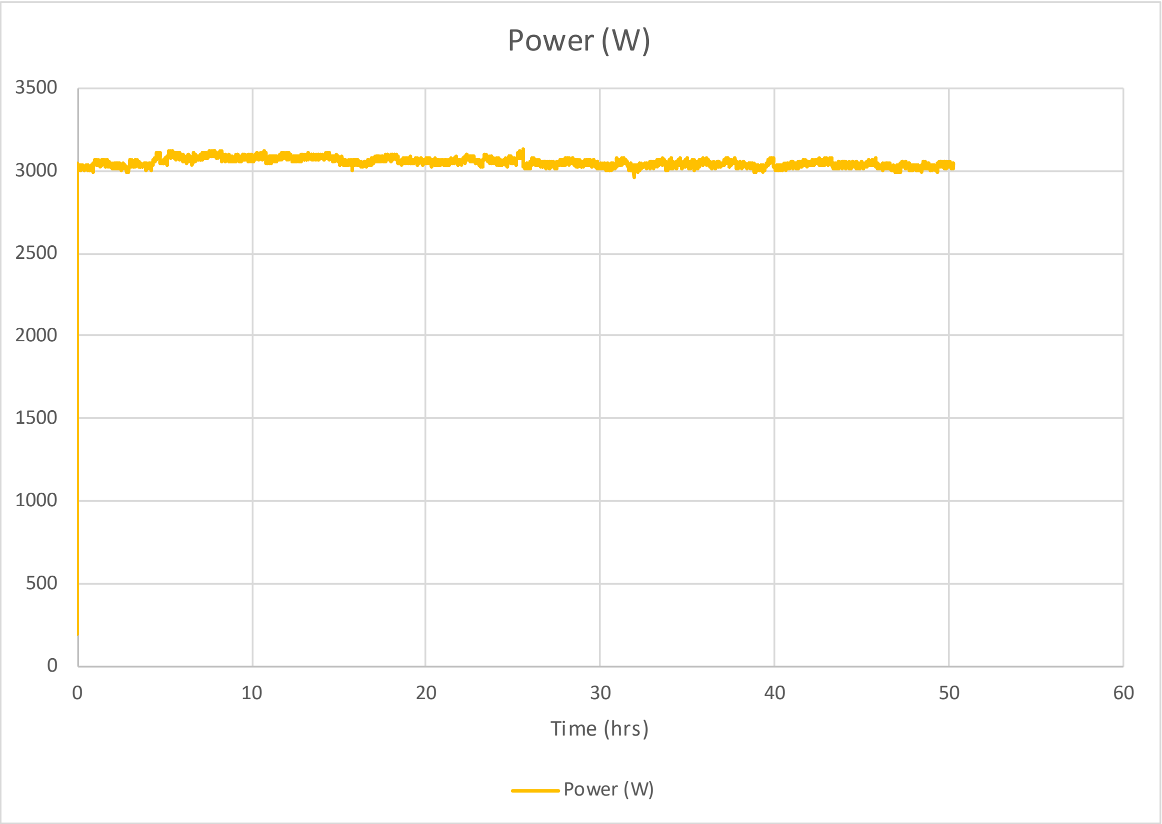
OKLAHOMA STATE UNIVERSITY

GSHP U

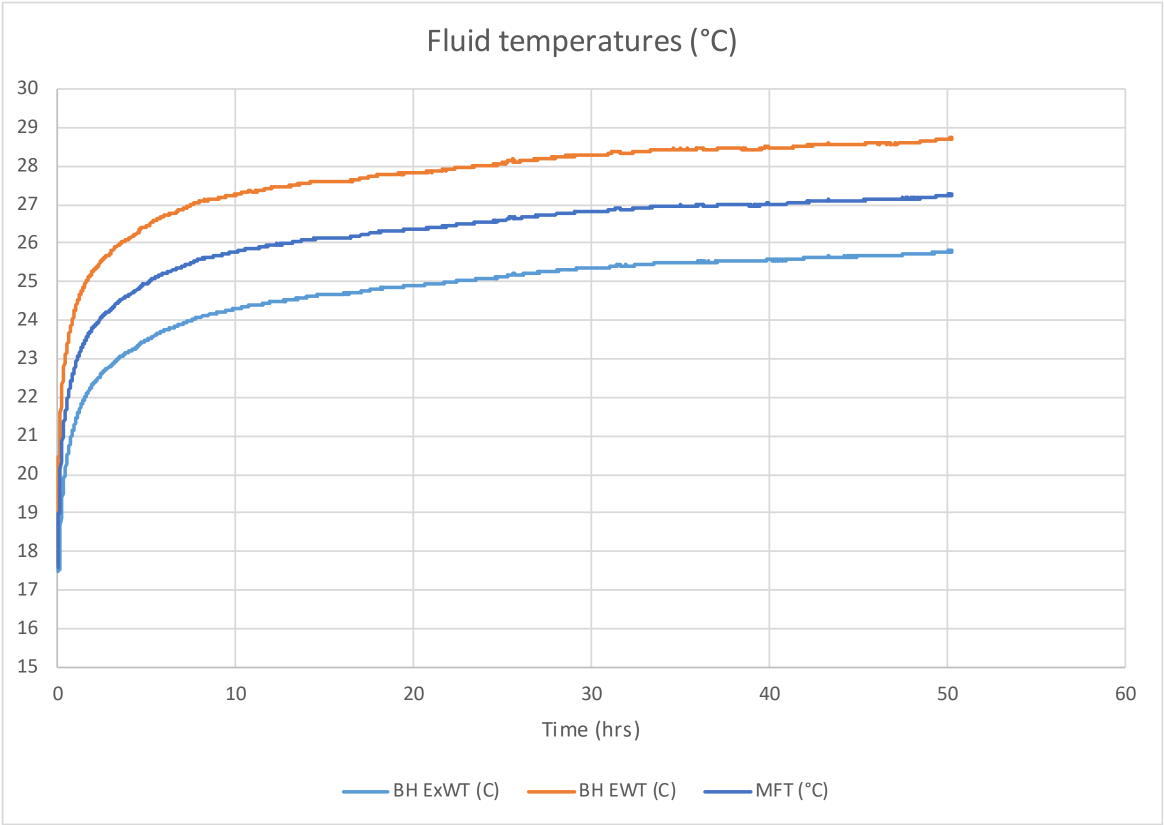
Example – Line source with temporal superposition

- Thermal response tests (TRT) are used to make in situ thermal conductivity measurements of the ground surrounding a borehole.
- A heat pulse is imposed on a U-tube and the temperature response is used to estimate the thermal conductivity.
- For a 1999 test, I've converted the one-minute measurements to hourly values.
- For analysis of TRT, parameter estimation techniques are often used to find k_{ground}, R_b that give best fit.
- I have NOT done this here. We simply have a simulation that we can use to look at the effects of changing k_{ground}, R_{b}.

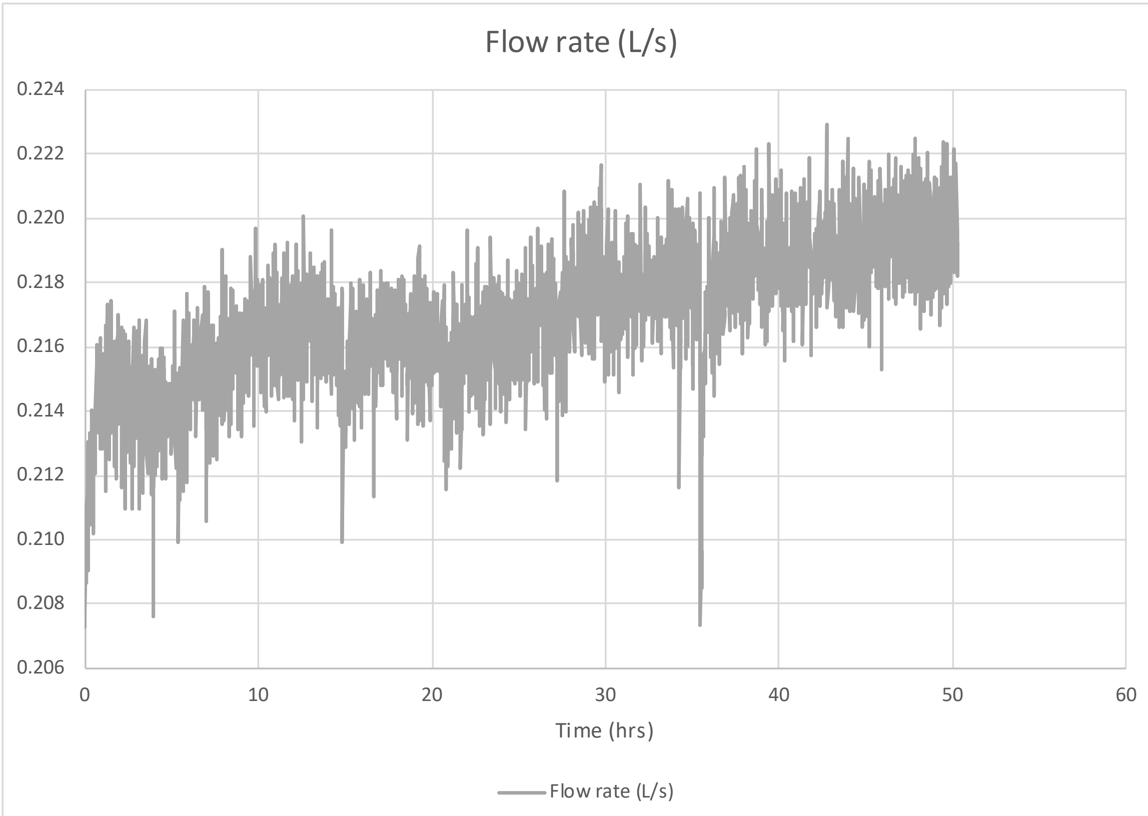
Measured results (minutely)



Measured results (minutely)



Measured results (minutely)



Simulation: VBA

Note: rho-cp here is too high by a factor of 10.

- Reads simulation parameters from "Sim" worksheet
- Reads hourly average power and hour-end flow rate from "Sim"worksheet.
- Simulate using VBA code in Line_source_example_1.xlsm

	H	I	J	K	L	M
	Simulation Parameters					
	UGT (°C)	17.53				
	k (W/(m·K)	2.8		Simulate		
	ρ-c_p (J/kgK)	25000000				
	R_b (K/ (W/m))	0.18				
	Depth (m)	76	(check)			
	Borehole rad (m	0.07	(check)			
					30.00	

10	Time (seconds)	Measured ExWT (C)	Measured EWT (C)	Flow rate (L/s)	Power (W)	Measured MFT (°C)	Flow rate (kg/s)
11	3600	21.41	24.34	0.216	3020.4	22.88	0.2151
12	7200	22.36	25.29	0.215	3044.8	23.83	0.2145
13	10800	22.83	25.78	0.215	3024.6	24.31	0.2140
14	14400	23.19	26.13	0.215	3037.2	24.66	0.2148
15	18000	23.49	26.44	0.213	3055.6	24.96	0.2121
16	21600	23.74	26.72	0.217	3089.3	25.23	0.2165
17	25200	23.91	26.88	0.215	3073.8	25.39	0.2145
18	28800	24.08	27.08	0.217	3095.6	25.58	0.2160
19	32400	24.19	27.16	0.215	3079.3	25.68	0.2144
20	36000	24.29	27.26	0.217	3076.5	25.77	0.2161
21	39600	24.39	27.33	0.219	3083.0	25.86	0.2186
22	43200	24.47	27.43	0.215	3079.6	25.95	0.2148
23	46800	24.53	27.48	0.217	3078.1	26.00	0.2164
24	50400	24.60	27.55	0.218	3082.9	26.08	0.2169

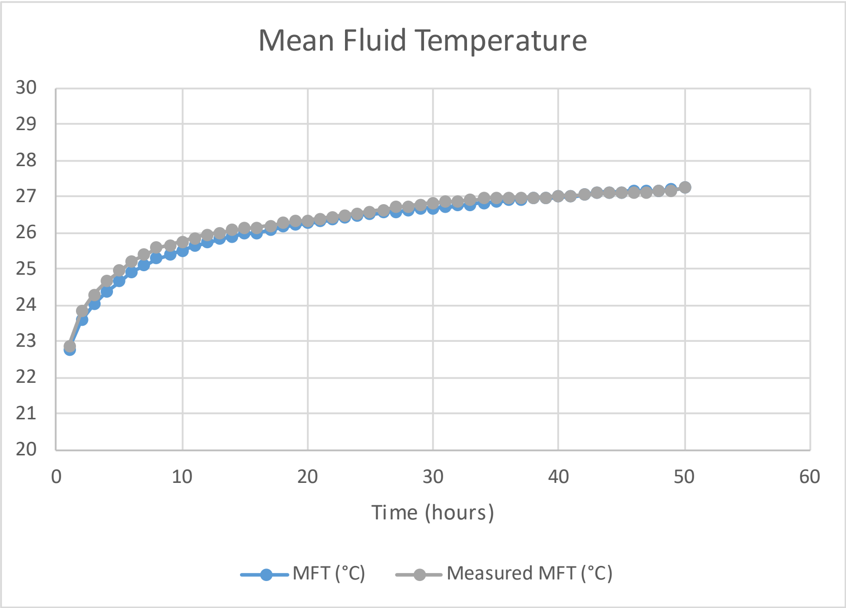
```
Sub SimGHE()
'JDS 2 April 2020
'Simulates one particular case with uniform time intervals
Dim q(100) As Double, mdot(100) As Double, ExWT(100) As Double, MFT(100) As Double
Dim alpha As Double, k As Double, rhocp As Double

q(0) = 0
tinterval = 3600 'seconds
'Read parameters
UGT = Worksheets("Sim").Cells(2, 9)
k = Worksheets("Sim").Cells(3, 9)
rhocp = Worksheets("Sim").Cells(4, 9)
Rb = Worksheets("Sim").Cells(5, 9)
Depth = Worksheets("Sim").Cells(6, 9)
BHradius = Worksheets("Sim").Cells(7, 9)

'compute intermediate values
Pi = 3.141592
Fpk = 4 * Pi * k
alpha = k / rhocp
rbsqoFas = (BHradius ^ 2) / (4 * alpha) 'Square of the borehole radius divided by 4 alpha

Nintervals = 50
'Read hourly values of heat transfer rate and fluid mass flow rate.
For i = 1 To Nintervals
   q(i) = Worksheets("Sim").Cells(10 + i, 5) / Depth
   mdot(i) = Worksheets("Sim").Cells(10 + i, 7)
   Next i
```

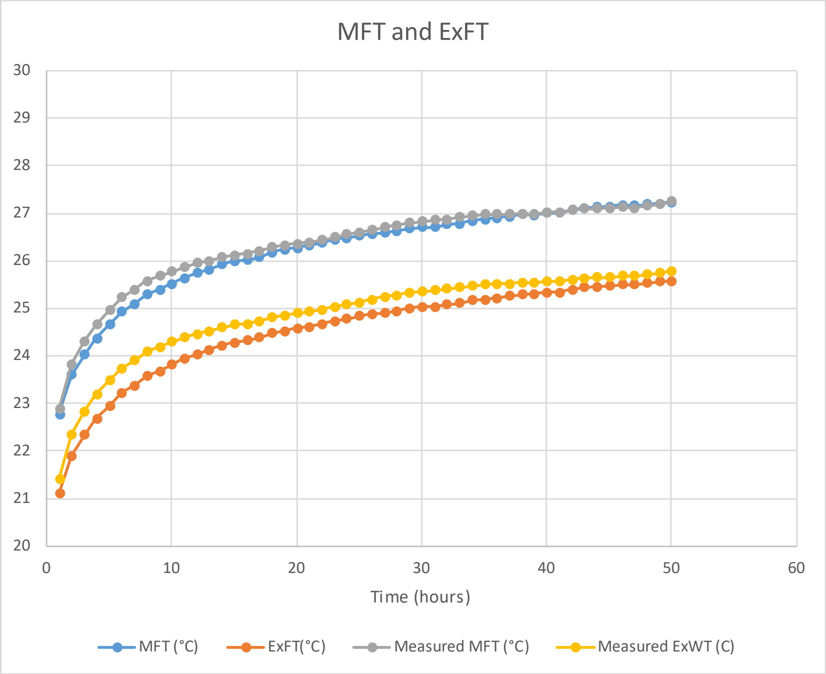
```
For n = 1 To Nintervals
   DTsum = 0
   For i = 1 To n
      dtime = tinterval * (n - i + 1) 'the time over which a specific pulse is applied.
      qpulse = q(i) - q(i - 1) 'magnitude of each devolved pulse
      z = rbsqoFas / dtime
      E1val = E1simple(z)
      DT = qpulse / Fpk * E1val 'Contribution of each pulse to the temperature change at the BH wall
      DTsum = DTsum + DT
      Next i
   MFT(n) = DTsum + q(n) * Rb + UGT 'Mean fluid temperature
   cpf = 1000 * WCP(MFT(n)) 'estimate cp based on computed mean temperature
   ExWT(n) = DTsum + q(n) * Rb + UGT - q(n) * Depth / (2 * mdot(n) * cpf) 'exiting fluid temperature
   nrow = n + 10
   Worksheets("Sim").Cells(nrow, 9) = MFT(n)
   Worksheets("Sim").Cells(nrow, 10) = ExWT(n)
   Next n
End Sub
```

```
Function E1simple(z)
'JDS 2 April 2020
' This function implements the expansion give in Abramowitz and Stegun
'See Equation 5.1.11
'M is the number of terms used (summation from n=1 to M instead of n=1 to infinity
Dim Sum As Double, n As Long
Dim gamma As Double
gamma = 0.5772156649
E1simple = -gamma + Log(1 / z)
End Function
```

Results

- With some adjustment to the inputs… (the whole point of the TRT!)





Thermal Response Tests

- Much more to be said about TRT.
- For overview, see Spitler and Gehlin(2015)

GHE often have more than one borehole!

- This requires the use of spatial superposition.
- E.g. for two boreholes, 5m apart:

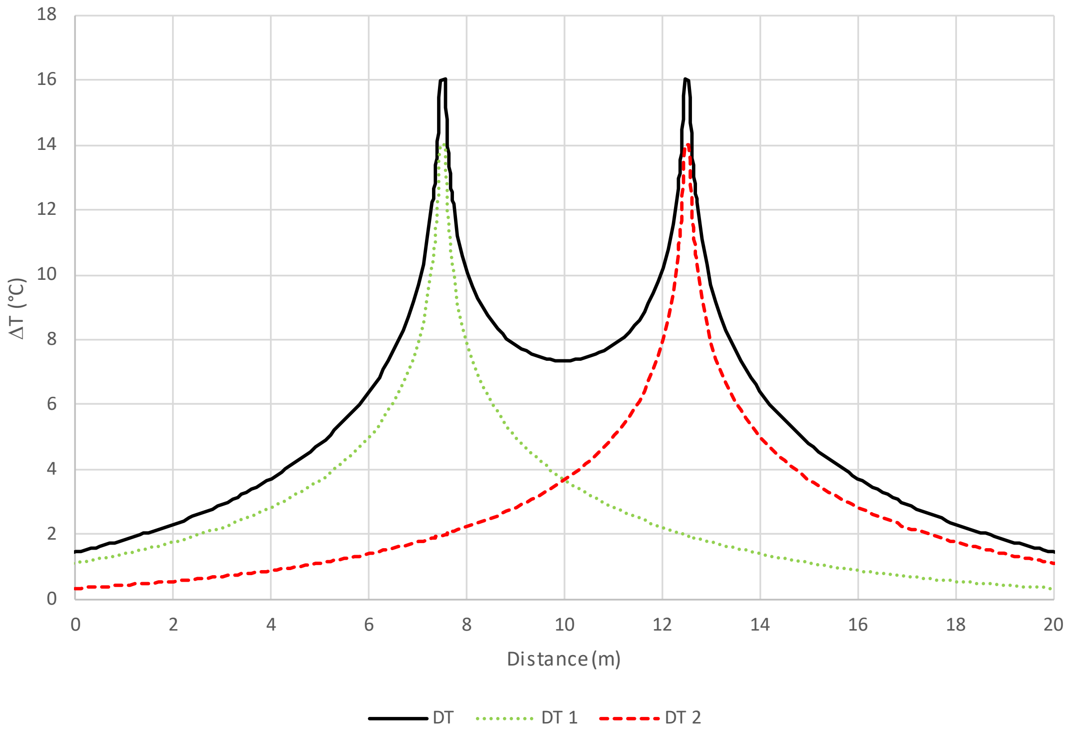


Figure originally drawn for Spitler and Bernier (2016) and is further discussed there.

Future lecture topics

- Using non-uniform time steps and why this is needed.
- Improving computational speed by aggregating loads.
- Non-dimensional response functions (g-functions)
- Computation of g-functions
- Boundary schemes used to compute g-functions.
- Thermal response tests.

References

- Many of the papers referenced below (authored by Dr. Spitler) are available at https://hvac.okstate.edu
- Software
  - GLHEPRO: https://hvac.okstate.edu/glhepro/overview
  - EED: https://buildingphysics.com/eed-2/
  - EnergyPlus: https://energyplus.net/
- Overview
  - Design methods: Spitler, J. D. and M. Bernier 2016. Vertical borehole ground heat exchanger design methods in Advances in Ground-Source Heat Pump Systems. S. Rees. Amsterdam, Woodhead Publishing: 29-61.
  - TRT: Spitler, J. D. and S. E. A. Gehlin. 2015. *Thermal response testing for ground source heat pump systems—An historical review*. Renewable and Sustainable Energy Reviews 50(0): 1125-1137.

References

Carslaw, H. S. and J. C. Jaeger 1959. Conduction of Heat in Solids, Second Edition. Oxford, Clarendon Press.

Claesson, J. and G. Hellström. 2011. *Multipole method to calculate borehole thermal resistances in a borehole heat exchanger*. HVAC&R Research 17(6): 895-911.

Claesson, J. and S. Javed. 2018. *Explicit Multipole Formulas for Calculating Thermal Resistance of Single U-Tube Ground Heat Exchangers*. Energies 11(1): 214.

Claesson, J. and S. Javed. 2019. *Explicit multipole formulas and thermal network models for calculating thermal resistances of double U-pipe borehole heat exchangers*. Science and Technology for the Built Environment 25(8): 980-992.

Eskilson, P. 1987. *Thermal Analysis of Heat Extraction Boreholes*. PhD Doctoral Thesis, University of Lund.

Eskilson, P. and J. Claesson. 1988. *Simulation Model for Thermally Interacting Heat Extraction Boreholes*. Numerical Heat Transfer 13(2): 149-165.

Gautschi, W. and W. F. Cahill 1964. Exponential Integral and Related Functions in Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. M. Abramowitz and I. Stegun. Washington, D.C., National Bureau of Standards.

Hellström, G. 1991. *Ground Heat Storage – Thermal Analyses of Duct Storage Systems - Theory* PhD Doctoral Thesis, University of Lund.

Ingersoll, L. R. and H. J. Plass. 1948. *Theory of the Ground Pipe Heat Source for the Heat Pump*. ASHVE Transactions 54: 339-348.

Paul, N. 1996. *The Effect of Grout Thermal Conductivity on Vertical Geothermal Heat Exchanger Design and Performance*. MS, South Dakota State University.

References

Spitler, J. D. and M. Bernier 2016. Vertical borehole ground heat exchanger design methods in Advances in Ground-Source Heat Pump Systems. S. Rees. Amsterdam, Woodhead Publishing: 29-61.

Spitler, J. D. and S. E. A. Gehlin. 2015. *Thermal response testing for ground source heat pump systems—An historical review*. Renewable and Sustainable Energy Reviews 50(0): 1125-1137.

Thomson, W. 1884. Compendium of the Fourier Mathematics for the Conduction of Heat in Solids, and the Mathematically Allied Physical Subjects of Diffusion of Fluids, and Transmission of Electric Signals through Submarine Cables. Article 72 in Mathematical and Physical Papers, Volume II. Cambridge, University Press: 41-60.

Xu, X. and J. D. Spitler. 2006. *Modelling of Vertical Ground Loop Heat Exchangers with Variable Convective Resistance and Thermal Mass of the Fluid*. 10th International Conference on Thermal Energy Storage - Ecostock 2006, Pomona, NJ. 8.

Yavuzturk, C. and J. D. Spitler. 1999. *A Short Time Step Response Factor Model for Vertical Ground Loop Heat Exchangers*. ASHRAE Transactions 105(2): 475-485.

Yavuzturk, C., J. D. Spitler and S. J. Rees. 1999. *A Transient Two-Dimensional Finite Volume Model for the Simulation of Vertical U-Tube Ground Heat Exchangers*. ASHRAE Transactions 105(2): 465-474.

Young, T. R. 2004 *Development, Verification, and Design Analysis of the Borehole Fluid Thermal Mass Model for Approximating Short Term Borehole Thermal Response*. MS, Oklahoma State University.