

This paper has been downloaded from the Building and Environmental Thermal Systems Research Group at Oklahoma State University ([www.hvac.okstate.edu](http://www.hvac.okstate.edu))

The correct citation for the paper is:

Spitler, J.D., D.E. Fisher. 1999. On The Relationship between the Radiant Time Series and Transfer Function Methods for Design Cooling Load Calculations. HVAC&R Research. Vol. 5, No. 2. pp.123-136.

Reprinted by permission from HVAC&R Research (Vol. #5, No. 2, pp. 123-136).  
© 1999 American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.

# On The Relationship between the Radiant Time Series and Transfer Function Methods for Design Cooling Load Calculations

**Jeffrey D. Spitler, Ph.D., P.E.**

Member ASHRAE

**Daniel E. Fisher, Ph.D., P.E.**

Member ASHRAE

---

*In practice, design cooling load calculations are based on steady periodic inputs, but historically cooling load procedures have not taken advantage of this fact. The generalized response factors, conduction transfer functions and room transfer functions currently used in cooling load methods impose an unnecessary computational burden on the procedure. This paper discusses the steady periodic nature of the cooling load calculation and develops the steady periodic conduction and thermal zone response factors used in the new Radiant Time Series Method (RTSM) that was introduced by Spitler et al. (1997). The periodic response factors used in the RTSM are compared to the aperiodic response factors used in the Transfer Function Method (TFM) described by McQuiston and Spitler (1992). Periodic response factors simplify the computational procedure and provide a means of comparing zone and surface types.*

---

## INTRODUCTION

Recently, a new method for performing design cooling load calculations, the Radiant Time Series Method (RTSM), has been developed to take advantage of the steady, periodic nature of design cooling load procedure input parameters. The RTSM, as described by Spitler, et al. (1997), converts the radiant portion of hourly heat gains to hourly cooling loads using radiant time factors, the coefficients of the radiant time series. Like response factors, radiant time factors are used to calculate the cooling load for the current hour on the basis of current and past heat gains. The radiant time series for a particular zone gives the time dependent response of the zone to a single pulse of radiant energy. The series shows the portion of the radiant pulse that is convected to the zone air for each hour.

The Radiant Time Series Method differs from the Transfer Function Method in the computation of conduction heat gain and in the determination of the cooling loads once the hourly heat gains are known. The TFM relies on a conduction transfer function to compute conduction heat gain and a room transfer function (weighting factors) to determine the cooling loads. The RTSM relies on a 24 term response factor series to compute conduction heat gain and another 24 term response factor series to determine the cooling loads.

The methods also differ fundamentally in the nature of the response factor excitation pulse. A steady, periodic excitation computationally simplifies the RTSM but limits the method to steady periodic (design day) inputs. Unlike the transfer function method, which results in a set of equations that must be solved iteratively, the periodic response factor based equations can be solved directly. Problems related to stability and convergence are avoided and for most cases computation time can be reduced. In addition, because periodic response factors operate directly on

---

**Jeffrey D. Spitler** is an associate professor at Oklahoma State University, Stillwater and **Daniel E. Fisher** is a senior research associate at the University of Illinois, Urbana-Champaign.

**Table 1. Nomenclature**

$Q$	cooling load, W	<b>B, D</b>	b and d CTF coefficient matrices
$q$	heat gain, W	$T_e$	column vector containing sol-air temperatures
$q''$	heat flux, W/m <sup>2</sup>	$S_b$	column vector containing the $t_{rc}\Sigma b_n$ products
$Z, Y$	conduction response factors	<b>Y</b>	periodic response factor matrix
$R$	room response factor	$S_y$	column vector containing the $t_{rc}\Sigma Y_n$ products
$r$	radiant time series response factors	<b>R</b>	radiant time series coefficient matrix
$v, w$	room transfer functions	<b>C<sub>1</sub>, C<sub>2</sub></b>	$v$ and $w$ room transfer function coefficient matrices
$b, d, c$	conduction transfer functions		
$T$	temperature, °C	<b>Subscripts</b>	
$A$	area, m <sup>2</sup>	$P$	periodic
Matrices		$j, k, l, n$	indices
$q''$	column vector containing conduction heat fluxes	$\theta$	time, h
$q$	column vector containing heat gains	$\delta$	time interval, h
$Q$	column vector containing cooling loads	$e$	sol-air
		$rc$	constant room air

either temperature differences or fluxes, the relationship between the series of coefficients and the building's construction may be more apparent.

The paper is organized in four sections. First, a framework for the development of periodic response factors is presented. This background section outlines the development of response factors and transfer functions used for transient conduction heat transfer and determination of cooling loads. Second, brief overviews of the RTSM and the TFM are presented. The third section develops the relationship between periodic wall response factors and conduction transfer functions, and the fourth section develops the relationship between the periodic radiant time series and room weighting factors. The nomenclature shown in Table 1 is used throughout the paper.

## BACKGROUND

Response factor-based and transfer function-based cooling load calculation procedures have been under development since the 1950s. Historically, they have been seen as a computationally feasible alternative to the heat balance approach, so called because it relies on a set of heat balances at each surface. Since both response factors and transfer functions are representations of an infinite series, they are necessarily approximations. They tend to differ in the number of excitation and response terms and in the nature of the excitation pulse. In general, there is a trade-off between the accuracy of the calculation and the computational requirements. In fact, many of the developments reported in the literature are aimed at improving the speed and/or accuracy of the calculation. A brief review of response factor-based and transfer function-based cooling load calculation procedures follows. First, a brief description of each method is in order.

Response factor methods relate the current value of the cooling load<sup>1</sup> to the current and past values of the heat gains:

<sup>1</sup> In design cooling load calculation terminology, "heat gain" is the instantaneous rate at which heat is transferred into or generated in the space. "Cooling load" is the rate at which heat must be removed from the zone to maintain a constant zone air temperature (ASHRAE 1997).

$$Q_{\theta} = \sum_{j=0}^J R_j q_{\theta-j\delta} \quad (1)$$

where  $J$  is the number of response factors actually used.

Response factor-based methods also use response factors to model transient heat conduction in walls and roofs:

$$q_{\theta}'' = \sum_{j=0}^J Y_j T_{e,\theta-j\delta} - \sum_{j=0}^J Z_j T_{rc,\theta-j\delta} \quad (2)$$

where  $J$  is a number, sometimes large, which depends on the construction of the wall.

Transfer function methods relate the current value of the cooling load to current and past values of the heat gains and past values of the cooling loads.

$$Q_{\theta} = \sum_{j=0}^J v_j q_{\theta-j\delta} - \sum_{k=0}^K w_k Q_{\theta-k\delta} \quad (3)$$

where  $v_j$  and  $w_k$  are the coefficients of the room transfer function, also known as weighting factors.

Similarly, transfer function methods generally use conduction transfer functions (CTFs) to model transient conduction heat transfer in walls and roofs:

$$q_{\theta}'' = \sum_{n=0} b_n T_{e,\theta-n\delta} - \sum_{n=1} d_n q_{\theta-n\delta}'' - T_{rc} \sum_{n=0} c_n \quad (4)$$

where each summation has as many terms as there are nonzero values of the coefficients. In design cooling load calculations the interior room temperature is usually assumed to be constant. If the room temperature was time-dependent, the room temperature would appear inside the last summation in Equation (4).

The first cooling load calculation procedure to make use of a digital computer was described by Briskin and Reque (1956). They used a simplified thermal circuit model of a house to obtain both wall and roof response factors and zone response factors. Wall and roof response factors were determined using a two-lump thermal circuit, a rectangular excitation pulse, and a Laplace transform-based solution to the two ordinary differential equations. The zone response factors were determined using the network properties of the entire thermal circuit.

A total of seven response factors were computed for both walls and roofs and for the entire zone. The methodology was applied both to the determination of peak heating and cooling loads and to the determination of cumulative heating and cooling loads (i.e. energy analysis). However, when the energy analysis was done, only six response factors were used, due to "computing machine limitations." It was noted that using six response factors instead of seven resulted in a 11.9% difference in predicted peak heat gain from the roof. However, no estimate was made of the error due to truncating an infinite series of response factors down to just seven terms.

Mitalas and Stephenson (1967) described a procedure for obtaining both wall and roof response factors and room thermal response factors. In contrast to Briskin and Reque, a triangular pulse was used to obtain the response factors, a more detailed thermal circuit was used to generate the zone response factors, and an exact analytical solution for the transient heat conduction problem was used to generate the wall and roof response factors.

A companion paper (Stephenson and Mitalas 1967) describes the use of the response factors in a cooling load calculation procedure. Unfortunately, the authors did not indicate how many wall/roof response factors or zone response factors were actually used in their calculations. Zone response factors are given for many of the heat gains. Ten response factors are given for each heat gain component. The ratio of successive terms (common ratio) is also given, so that the length of the response factor series may be adjusted to achieve the desired accuracy.

Later, Stephenson and Mitalas (1971) presented a transfer function approach to modeling transient heat conduction in multi-layer slabs. Two methods for determining the transfer function coefficients were presented: the first based on using an excitation function with known Laplace-transform and z-transform, and the second based on matching the frequency response to the frequency response of the s-transfer function at several frequencies. The authors imply the motivation for developing the z-transfer functions is that "... they are much more economical in terms of computer memory space and running time." However, they do not quantify the savings in computer time or memory, nor do they state explicitly how the number of response factors required to achieve a certain accuracy compares to the number of transfer function coefficients. In a reply to a question asked in the discussion, one of the authors estimates that there is a five-fold reduction in the number of arithmetic operations required when using the CTF formulation when compared to using the response factor formulation.

In 1972, apparently without ever being published in a peer-reviewed archival publication, the transfer function method for computing zone thermal response was introduced in the ASHRAE Handbook of Fundamentals (ASHRAE 1972). The procedure for obtaining the room transfer function coefficients was not documented in the handbook, but an undated computer program written at the National Research Council of Canada was cited. The method, as presented in the handbook, relied on a set of tabulated room transfer function coefficients. For each heat gain component, four heat gain coefficients ( $v$ ) and three cooling load coefficients ( $w$ ) were tabulated.

Kerrisk et al. (1981) present the custom weighting factor routine used in the DOE 2 energy analysis program. Initially, a set of response factors were determined by exciting a heat balance model with a unit heat gain pulse. They explain that while the response factor formulation could be used, "... (it) would require storage for a large number of past values of the excitation function (input) and completion of a long sum at each hour of the simulation." Consequently, they developed a transfer function of the form:

$$Q_{\theta} = \sum_{j=0}^2 v_j q_{\theta-j\delta} - \sum_{k=1}^2 w_k Q_{\theta-k\delta} \quad (5)$$

The coefficients were determined by setting the z-transform forms of the response factor formulation and transfer function formulation equal to each other:

$$d_0 + d_1 z^{-1} + d_2 z^{-2} + \dots = \frac{v_0 + v_1 z^{-1} + v_2 z^{-2}}{1 + w_1 z^{-1} + w_2 z^{-2}} \quad (6)$$

By combining coefficients of like powers of  $z^{-1}$ , an infinite set of equations can be formed. The authors used five of the equations to solve for the  $v$  and  $w$  coefficients. They note that it is an approximation, but that it "... has been found to be adequate for weighting factors." Beyond that, the authors did not attempt to characterize the error associated with using the more compact transfer function form.

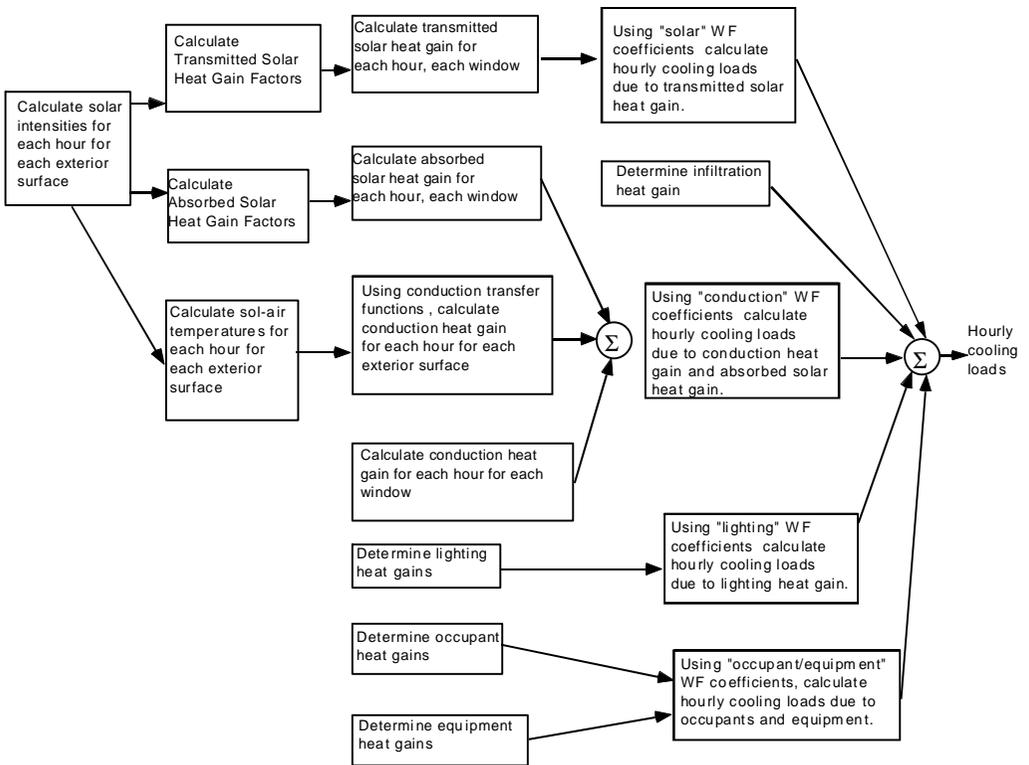
The formulation described by Kerrisk, et al. (1981) is the basis for the TFM procedure currently recommended by ASHRAE (1997). However, a series of ASHRAE research projects resulted in a new procedure for determining weighting factors (Sowell 1988a, 1988b, 1988c) and CTF coefficients (Harris and McQuiston 1988).

To summarize, the weighting factors described by Equation (5) and the conduction transfer function described in Equation (4) form the basis of the TFM. None of the formulations of the response factor methods and transfer function methods described above take advantage of the steady periodic nature of the design day cooling load calculation. The RTSM, although based on response factors for both conduction heat transfer (Equation 2) and for the conversion of heat gains to cooling loads (Equation 1), takes advantage of the steady periodic nature of the calculation by using steady periodic excitation functions to develop the response factor coefficients.

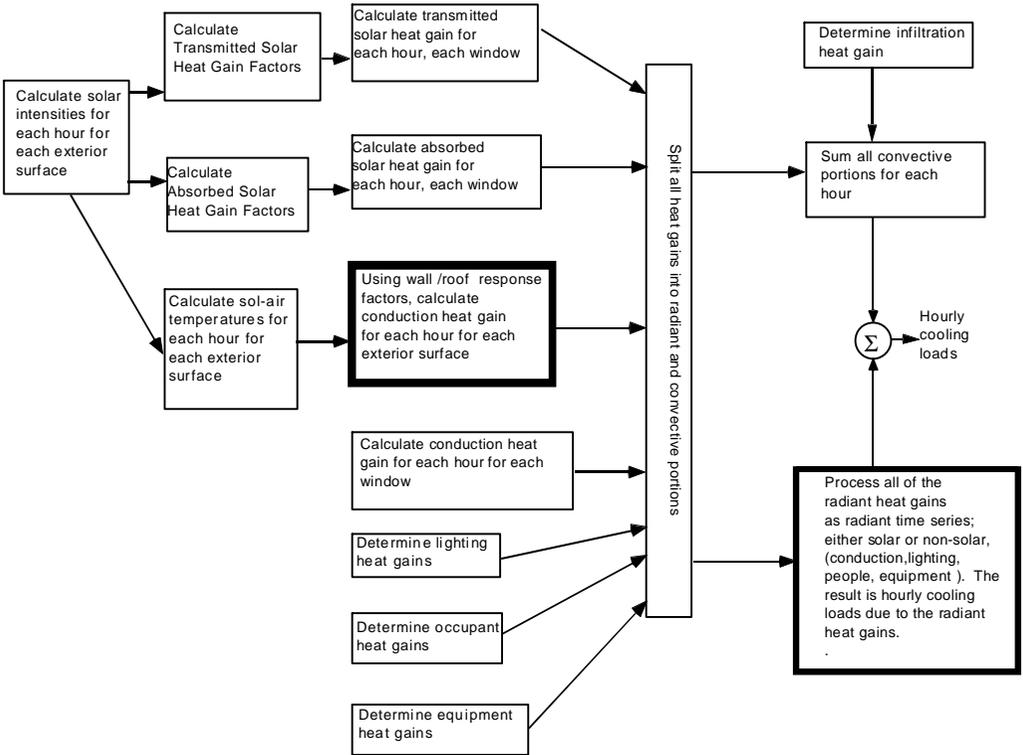
**OVERVIEW OF THE RTSM AND THE TFM**

In many respects, the Radiant Time Series Method follows the Transfer Function Method described by McQuiston and Spitler (1992) and ASHRAE (1997). Figures 1 and 2 illustrate the computational procedure of the TFM and RTSM respectively. A comparison of the figures shows that the calculation of solar radiation, transmitted solar heat gain through windows, solar heat gain absorbed by windows, sol-air temperature, and infiltration are exactly the same in both methods.

The TFM uses conduction transfer functions to calculate conduction heat gains as shown in Figure 1. The corresponding box in Figure 2 shows that the RTSM uses periodic response



**Figure 1. Overview of the Transfer Function Method (McQuiston and Spitler, 1992)**



**Figure 2. Overview of the Radiant Time Series Method (Spitler, et al. 1997)**

factors to calculate conduction heat gains. In addition, the TFM applies weighting factors to the combined convective and radiant heat gains as shown in Figure 1. The radiant time series (shown in Figure 2) operates only on the radiative portion of the gains. Although the convective portion of the heat gain is included in the room transfer function, this is not a defining feature of the room transfer function. The significant difference between the methods is the use of periodic response factors in the RTSM.

The radiant time series method takes advantage of the fact that design cooling load calculations are typically performed for a single, 24 hour, peak design day. Since some history is required, the RTSM assumes that previous days are identical to the peak design day. The 24 hourly cooling load values calculated for the peak design day represent a steady periodic response to a steady periodic input of 24 hourly heat gains.

## PERIODIC RESPONSE FACTORS FOR SURFACE CONDUCTION

The periodic response factor formulation gives a time series solution to the transient, one-dimensional conduction heat transfer problem. For any hour  $\theta$ , the conduction heat gain for the surface,  $q_{\theta}$ , is given by the summation of the periodic response factors multiplied by the temperature difference across the surface as shown in Equation (7).

$$q_{\theta}'' = \sum_{j=0}^{23} Y_{Pj} T_{e, \theta - j\delta} - T_{rc} \sum_{j=0}^{23} Z_{Pj} \quad (7)$$

The standard CTF formulation is shown in Equation (4). Unlike the response factor formulation, which only requires a temperature history, the CTF formulation requires both a temperature history and a heat flux history.

**Relationship of Periodic Response Factors to Conduction Transfer Functions**

The relationship between conduction transfer functions and periodic response factors can be established by deriving the response factors directly from a steady periodic formulation of the standard CTF equation. This approach defines the steady periodic response factors in terms of conduction transfer functions. Noting that  $\Sigma b_n = \Sigma c_n$ , the general CTF Equation (4) can be written for each hour to form a set of 24 hourly equations, as follows:

$$q''_1 = \sum_{n=0} b_n T_{e, 1-n\delta} - \sum_{n=1} d_n q''_{1-n\delta} - T_{rc} \sum_{n=0} b_n \tag{8}$$

$$q''_2 = \sum_{n=0} b_n T_{e, 2-n\delta} - \sum_{n=1} d_n q''_{2-n\delta} - T_{rc} \sum_{n=0} b_n \tag{8a}$$

...

$$q''_{24} = \sum_{n=0} b_n T_{e, 24-n\delta} - \sum_{n=1} d_n q''_{24-n\delta} - T_{rc} \sum_{n=0} b_n \tag{8b}$$

The 24 hourly equations can be rearranged and written in the following matrix form:

$$\begin{pmatrix} 1 & 0 & 0 & \dots & d_3 & d_2 & d_1 \\ d_1 & 1 & 0 & 0 & \dots & & d_2 \\ d_2 & d_1 & 1 & 0 & 0 & \dots & \\ & & & \dots & & & \\ & & & & \dots & & \\ & & & & & \dots & \\ & & & & & & \dots & d_3 & d_2 & d_1 & 1 \end{pmatrix} \cdot \begin{pmatrix} q''_1 \\ q''_2 \\ q''_3 \\ \dots \\ q''_{24} \end{pmatrix} = \begin{pmatrix} b_0 & 0 & 0 & \dots & & b_2 & b_1 \\ b_1 & b_0 & 0 & 0 & \dots & & b_2 \\ b_2 & b_1 & b_0 & 0 & 0 & \dots & \\ & & & \dots & & & \\ & & & & \dots & & \\ & & & & & \dots & \\ & & & & & & b_4 & b_3 & b_2 & b_1 & b_0 \end{pmatrix} \cdot \begin{pmatrix} T_{e1} \\ T_{e2} \\ T_{e3} \\ \dots \\ T_{e24} \end{pmatrix} - \begin{pmatrix} T_{rc} \Sigma b_n \\ \dots \\ T_{rc} \Sigma b_n \end{pmatrix} \tag{9}$$

Equation (9) can be rearranged and represented more simply as:

$$\mathbf{q}'' = \mathbf{D}^{-1} \mathbf{B} \mathbf{T}_e - \mathbf{D}^{-1} \mathbf{S}_b \tag{10}$$

where

- D** = left hand side coefficient matrix
- q''** = column vector containing the conduction heat fluxes
- B** = right hand side coefficient matrix
- T<sub>e</sub>** = column vector containing the sol-air temperatures
- S<sub>b</sub>** = column vector containing the  $T_{rc} \Sigma b_n$  products

Noting that  $\Sigma Y_n = \Sigma Z_n$ , the periodic response factor Equation (7) can be written as a matrix formulation:

$$\begin{pmatrix} q_1'' \\ q_2'' \\ q_3'' \\ \dots \\ q_{24}'' \end{pmatrix} = \begin{pmatrix} Y_0 & Y_{23} & Y_{22} & \dots & Y_2 & Y_1 \\ Y_1 & Y_0 & Y_{23} & Y_{22} & \dots & Y_2 \\ Y_2 & Y_1 & Y_0 & Y_{23} & Y_{22} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ Y_2 & Y_1 & Y_0 & \dots & \dots & \dots \end{pmatrix} \bullet \begin{pmatrix} T_{e1} \\ T_{e2} \\ T_{e3} \\ \dots \\ T_{e24} \end{pmatrix} - \begin{pmatrix} T_{rc}\Sigma Y_n \\ \dots \\ \dots \\ \dots \\ T_{rc}\Sigma Y_n \end{pmatrix} \quad (11)$$

Equation (11) can be represented as:

$$\mathbf{q}'' = \mathbf{Y}\mathbf{T}_e - \mathbf{S}_y \quad (12)$$

where

- $\mathbf{q}''$  = column vector containing the conduction heat fluxes
- $\mathbf{Y}$  = periodic response factor matrix
- $\mathbf{T}_e$  = column vector containing the sol-air temperatures
- $\mathbf{S}_y$  = column vector containing the  $T_{rc}\Sigma Y_n$  products

Equations (10) and (12) can be combined to give:

$$\mathbf{D}^{-1}\mathbf{B}\mathbf{T}_e - \mathbf{D}^{-1}\mathbf{S}_b = \mathbf{Y}\mathbf{T}_e - \mathbf{S}_y \quad (13)$$

Equation (13) can be simplified by establishing that  $\mathbf{D}^{-1}\mathbf{S}_b = \mathbf{S}_y$  or, more conveniently, that  $\mathbf{S}_b = \mathbf{D}\mathbf{S}_y$ .  $\mathbf{S}_b$  is a column vector for which each element is the product  $T_{rc}\Sigma b_n$ .  $\mathbf{D}\mathbf{S}_y$  is a column vector for which each element is  $(1 + \Sigma d_n)T_{rc}\Sigma Y_n$ . The U-factor for a wall or roof may be expressed in terms of either periodic response factors or CTF coefficients:

$$U = \Sigma Y_n = \Sigma b_n / (1 + \Sigma d_n) \quad (14)$$

it follows that:

$$T_{rc}\Sigma b_n = (1 + \Sigma d_n)T_{rc}\Sigma Y_n \quad (15)$$

and that:

$$\mathbf{S}_b = \mathbf{D}\mathbf{S}_y \quad (16)$$

Therefore, Equation (13) may be further simplified to give the following relationship between conduction transfer function coefficients and periodic response factors:

$$\mathbf{Y} = \mathbf{D}^{-1}\mathbf{B} \quad (17)$$

This development defines the relationship between conduction transfer functions and the periodic response factor and graphically illustrates an important aspect of the CTF formulation. When driven by a steady, periodic input, the CTF formulation needlessly includes flux terms on the right hand side of the equation. With the above derivation, it is clear that the conduction heat gain can be determined with only a temperature history. For steady periodic inputs, both the CTF formulation and the periodic response factor formulation will give identical results.

### Comparison of Periodic Response Factors With Conduction Transfer Functions

In the CTF formulation, the response to periodic inputs is approached indirectly by updating the flux histories in successive calculations until the solution converges on the steady periodic response. The CTF formulation must be driven to the steady periodic condition each time it is applied. On the other hand, the periodic response factor approach pre-calculates part of the solution by using a periodic excitation pulse to generate the response factor series. As a result, the conduction heat gains may be computed directly.

The perspicuity of the steady periodic response factor formulation can be illustrated by comparing the CTFs and response factors for a standard wall construction, wall type 20 from Harris and McQuiston (1988). The wall is made up of an outside surface resistance, a finish layer, 15 mm insulation, 300 mm heavyweight concrete, and inside surface resistance. The CTF coefficients for this wall are shown in Table 2 and the response factors are shown in Table 3.

The CTF coefficients bear no obvious physical relationship to the wall thermal response. In contrast, a plot of the response factors (Figure 3a) shows the 7 hour lag between the input pulse and the peak response factor, which might be expected from such a massive wall. Unlike the CTFs, the response factors do provide some physical insight into the steady periodic response to a steady periodic unit sol-air temperature pulse. For comparison purposes, the response factors for a lightweight wall, wall type 3, have also been plotted in Figure 3b.

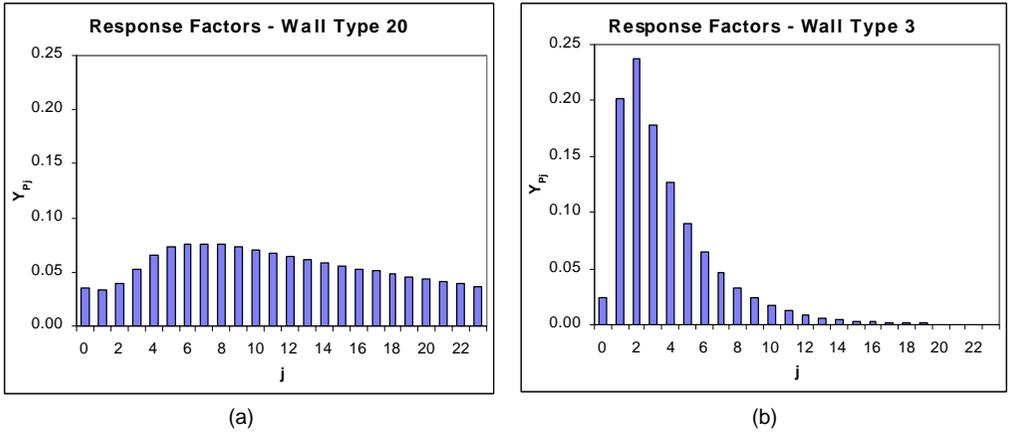
Four conclusions can be immediately drawn from the comparison. First, the periodic response factor formulation is computationally less intensive than the CTF formulation. The CTF formulation requires iteration over a number of days to reach steady periodic conditions. The number

**Table 2. CTF Coefficients for Wall Type 20**

$n$	$b_n$	$c_n$	$d_n$
0	0.00000		1.00000
1	0.00070		-1.86030
2	0.00677		1.05930
3	0.00873		-0.19508
4	0.00218		0.01002
5	0.00011		-0.00016
6	0.00000		0.00000
Sum	0.01849	0.01849	0.01378

**Table 3. Response Factors for Wall Type 20**

$Y_0$	0.035357	$Y_{12}$	0.064911
$Y_1$	0.034277	$Y_{13}$	0.061909
$Y_2$	0.039959	$Y_{14}$	0.058959
$Y_3$	0.053287	$Y_{15}$	0.056094
$Y_4$	0.065319	$Y_{16}$	0.053335
$Y_5$	0.072634	$Y_{17}$	0.050690
$Y_6$	0.075929	$Y_{18}$	0.048163
$Y_7$	0.076524	$Y_{19}$	0.045753
$Y_8$	0.075450	$Y_{20}$	0.043459
$Y_9$	0.073392	$Y_{21}$	0.041277
$Y_{10}$	0.070786	$Y_{22}$	0.039202
$Y_{11}$	0.067904	$Y_{23}$	0.037230



**Figure 3. Response Factors for Type 20 Wall and Type 3 Wall**

of days varies depending on the thermal mass of the wall or roof, but five or more days are often required. A comparison of Equations (9) and (11) shows that the CTF formulation, with two or more daily iterations, requires more mathematical operations than the periodic response factor formulation.

Second, solution of the CTF equation requires logical constructs (i.e. checking for convergence) that are best supported by a high-level computer language. Since the periodic response factor equation requires neither iteration, nor convergence checks, it can be solved directly and conveniently on a spreadsheet.

Third, the periodic response factor equation captures the form of the solution in a single series. This series, of necessity, completely expresses the physical response of the surface. As such, it has utility as an indicator of surface construction performance. The CTF equation, on the other hand, requires three sets of coefficients that operate on both fluxes and temperatures. The coefficients alone have no obvious utility as indicators of physical performance. Finally, the form of the periodic response factor equation is immediately recognizable as a heat flux calculation. This fact has both pedagogical and practical value. Practically, errors in both the formulation of the equation and in the coefficients themselves are much easier to identify in the periodic response factor equation.

### PERIODIC RESPONSE FACTORS FOR THERMAL ZONE RESPONSE

The transfer function method uses a room transfer function to relate the cooling load ( $Q$ ) for the current hour ( $\theta$ ) to the current heat gain ( $q_\theta$ ) and past values of the heat gain as shown in Equation (5). The cooling load for the current hour is dependent not only on heat gains and weighting factors ( $v$  and  $w$ ), but it also depends on cooling loads from previous hours.

For design cooling load calculations, the time interval ( $\delta$ ) is chosen to be one hour, so that  $Q_{\theta-\delta}$  represents the cooling load one hour ago, for  $\delta = 1$ . For the first and second hours of the day, values are required from the previous day. Since the single design day is assumed to repeat, heat gains and cooling loads for the 23rd and 24th hours of the previous day are equivalent to those for the 23rd and 24th hours of the current day, e.g.:

$$Q_1 = v_0 q_1 + v_1 q_{24} + v_2 q_{23} - w_1 Q_{24} - w_2 Q_{23} \quad (18)$$

Since the cooling loads for the 23rd and 24th hours of the previous day are not known at the beginning of the analysis, an assumption needs to be made for an initial history. Then, the

cooling loads for the single design day are calculated in an iterative manner, improving the estimate of the cooling load history each time, until some steady periodic convergence is reached.

The cooling load can also be expressed in response factor form as shown in Equation (19):

$$Q_{\theta} = r_0 q_{\theta} + r_1 q_{\theta-\delta} + r_2 q_{\theta-2\delta} + r_3 q_{\theta-3\delta} + \dots + r_{23} q_{\theta-23\delta} \tag{19}$$

In this formulation, the steady periodic zone response factors (the  $r$  terms) are the coefficients of the radiant time series, so called because the response factors operate only on the radiant portion of the hourly heat gains. Thus,  $r_0$  represents the fraction of the radiant pulse convected to the zone air in the current hour,  $r_1$  in the last hour, and so on.

Applying a procedure analogous to that described in Section 4.1, the zone transfer function formulation can be expressed as a matrix equation:

$$\begin{pmatrix} 1 & 0 & 0 & \dots & w_2 & w_1 \\ w_1 & 1 & 0 & 0 & \dots & w_2 \\ w_2 & w_1 & 1 & 0 & 0 & \dots \\ & & & \dots & & \\ & & & & w_2 & w_1 & 1 \end{pmatrix} \cdot \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ \dots \\ Q_{24} \end{pmatrix} = \begin{pmatrix} v_0 & 0 & 0 & \dots & v_2 & v_1 \\ v_1 & v_0 & 0 & 0 & \dots & v_2 \\ v_2 & v_1 & v_0 & 0 & 0 & \dots \\ & & & \dots & & \\ & & & & 0 & 0 & v_2 & v_1 & v_0 \end{pmatrix} \cdot \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ \dots \\ q_{24} \end{pmatrix} \tag{20}$$

or 
$$\mathbf{Q} = \mathbf{C}_1^{-1} \mathbf{C}_2 \mathbf{q} \tag{21}$$

where  $\mathbf{Q}$  is the column vector containing the cooling loads,  $\mathbf{C}_1$  is the  $v$  coefficient matrix,  $\mathbf{C}_2$  is the  $w$  coefficient matrix, and  $\mathbf{q}$  is the column vector containing the heat gains. Likewise the radiant time series formulation can be expressed as:

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ \dots \\ Q_{24} \end{pmatrix} = \begin{pmatrix} r_0 & r_{23} & r_{22} & \dots & r_2 & r_1 \\ r_1 & r_0 & r_{23} & r_{22} & \dots & r_2 \\ r_2 & r_1 & r_0 & r_{23} & r_{22} & \dots \\ & & & \dots & & \\ & & & & r_2 & r_1 & r_0 \end{pmatrix} \cdot \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ \dots \\ q_{24} \end{pmatrix} \tag{22}$$

or 
$$\mathbf{Q} = \mathbf{R} \mathbf{q} \tag{23}$$

where  $\mathbf{R}$  is the radiant time series coefficient matrix.

As with conduction transfer functions and steady periodic conduction response factors, the steady periodic radiant time series can be defined in terms of the zone weighting factors by combining Equations (21) and (23).

$$\mathbf{R} = \mathbf{C}_1^{-1} \mathbf{C}_2 \tag{24}$$

Once again, the advantages of a non-iterative procedure and physical insight into the zone response are realized by the response factor formulation for the special case of steady periodic inputs. From a pedagogical perspective, the identical form of the conduction response factor formulation and the zone response factor formulation also has some value.

Again, the radiant time series approach eliminates the need for a convergence check. The zone weighting factor approach will vary in the number of days required to reach steady periodic conditions. For a typical case requiring five daily iterations, the weighting factor approach will require approximately the same number of mathematical operations as the radiant time series approach.

The zone described in Example 2.1 of the *Cooling and Heating Load Calculation Manual* (McQuiston and Spitler 1992) is used to illustrate the procedure. The following room transfer function coefficients (weighting factors) are obtained for lighting heat gain:

$$\begin{aligned}v_0 &= 0.61116 \\v_1 &= -0.67267 \\v_2 &= 0.12987 \\w_1 &= -1.19718 \\w_2 &= 0.26554\end{aligned}$$

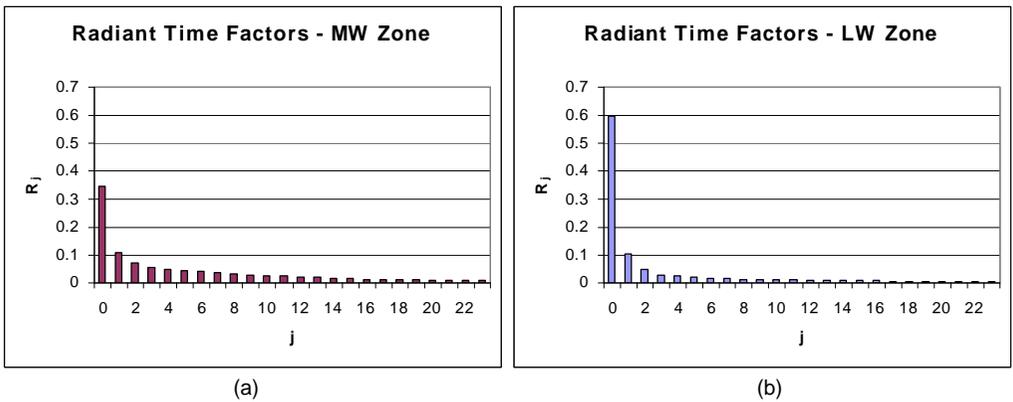
In the form above, the heat gain is assumed to be 59% radiative and 41% convective. The weighting factors for the radiative portion of the heat gain, are corrected using the procedure described in section A.5 of McQuiston and Spitler (1992). The corrected weighting factors are:

$$\begin{aligned}v_0 &= 0.340949 \\v_1 &= -0.30818 \\v_2 &= 0.035591 \\w_1 &= -1.19718 \\w_2 &= 0.26554\end{aligned}$$

Using the procedure described above, the weighting factors were transformed into a set of 24 radiant time factors shown in Figure 4a. The example zone might be thought of as a medium weight zone with the characteristic parameters described in Table 4. (For full definition of

**Table 4. Zone parameters for Mediumweight Zone and Lightweight Zone**

Zone Parameter	Mediumweight Zone	Lightweight Zone
Zone geometry	30.5 m × 6.1 m	30.5 m × 6.1 m
Zone height	3.7 m	3.7 m
Number of exterior walls	2	2
Interior shade	0%	0%
Furniture	Yes	Yes
Exterior construction	#4, heavyweight with moderate insulation	#1, lightweight with light insulation
Partition type	#1, lightweight	#1, lightweight
Zone location	Single-story	Single-story
Slab type	Slab-on-grade	Slab-on-grade
Ceiling type	With suspended ceiling	With suspended ceiling
Roof type	#2, heavyweight with no insulation	#1, lightweight with light insulation
Floor covering	vinyl tile	carpet with pad
Glass percent	10%	50%



**Figure 4. Radiant Time Factors for Medium Weight and Lightweight Zones**

parameter types, see McQuiston and Spitler (1992).) For comparison purposes, a lightweight zone was created, and its weighting factors were transformed into a set of 24 radiant time factors shown in Figure 4b. A comparison of Figures 4a and 4b demonstrates the physical significance of the radiant time factors. The lightweight zone has a much quicker response, shown in a much larger value of  $R_0$ , and smaller values of  $R_1$ – $R_{23}$ , when compared to the medium weight zone.

## CONCLUSIONS

Hourly design cooling loads are typically calculated using steady periodic inputs, and periodic response factors for conduction heat transfer and thermal zone response may be advantageously utilized in the computational procedure. General transfer function formulations must be driven to the final steady periodic response by an iterative procedure. Periodic response factors, on the other hand, calculate the steady periodic response directly. Although the periodic response factors for transient conduction heat transfer and thermal zone response are uniquely defined, the computational procedure is the same for both.

Four conclusions may be drawn from the comparison of transfer functions with periodic response factors. First, the periodic response factor formulation is, in general, computationally less intensive than the transfer function formulation. Conduction transfer functions always require more mathematical operations than steady periodic response factors. Zone weighting factors will require more mathematical operations than radiant time series if the number of daily iterations exceeds five.

Second, transfer function methods require an iterative procedure that is best implemented in a computer program with due consideration given to convergence and stability criteria. For design day calculations, the complexity of the transfer function implementation does not add anything to the rigor of the procedure. The periodic response factor equation can be solved directly and conveniently on a spreadsheet.

Third, periodic response factor equations capture the form of the solution in a single series. Transfer function equations, on the other hand, require coefficients that operate on both temperatures (or heat gains) and fluxes (or cooling loads). The single periodic response factor series completely expresses the physical response of the surface or zone. The entire zone or surface response is captured in a single set of coefficients, which have utility as a performance indicator. As a result, both the periodic conduction response factors and the periodic zone response factors provide some insight into the physics of the thermal process being modeled.

Finally, the form of the periodic response factor heat conduction equation is immediately recognizable as a heat flux calculation. The RTS equation is also immediately recognizable as a calculation that converts heat gains to a cooling load. This fact is advantageous both in the classroom and, as a practical matter, in identifying errors in the formulation of the equations and in the series themselves.

## REFERENCES

- ASHRAE. 1972. *ASHRAE Handbook, Fundamentals*. Atlanta: ASHRAE.
- ASHRAE. 1997. *ASHRAE Handbook, Fundamentals*. Atlanta: ASHRAE.
- Brisken, W.R., and S.G. Reque. 1956. Heat Load Calculations by Thermal Response. *ASHAE Transactions* 62: 391-424.
- Harris, S.M., and F.C. McQuiston. 1988. A Study to Categorize Walls and Roofs on the Basis of Thermal Response. *ASHRAE Transactions* 94(2): 688-714.
- Kerrisk, J.F., N.M. Schnurr, J.E. Moore, and B.D. Hunn. 1981. The Custom Weighting-Factor Method for Thermal Load Calculations in the DOE-2 Computer Program. *ASHRAE Transactions* 87(2): 569-584.
- McQuiston, F.C., and J.D. Spitler. 1992. *Cooling and Heating Load Calculation Manual*, 2nd Edition. Atlanta: ASHRAE.
- Mitalas, G.P., and D.G. Stephenson. 1967. Room Thermal Response Factors. *ASHRAE Transactions* 73(1): 2.1-2.10.
- Sowell, E.F. 1988a. Classification of 200,640 Parametric Zones for Cooling Load Calculations. *ASHRAE Transactions* 94(2): 754-777.
- Sowell, E.F. 1988b. Cross-Check and Modification of the Doe-2 Program for Calculation of Zone Weighting Factors. *ASHRAE Transactions* 94(2): 737-753.
- Sowell, E.F. 1988c. Load Calculations for 200,640 Zones. *ASHRAE Transactions* 94(2): 716-736.
- Spitler, J.D., D.E. Fisher, C.O. Pedersen. 1997. The Radiant Time Series Cooling Load Calculation Procedure. *ASHRAE Transactions* 103(2): 503-515.
- Stephenson, D.G. and G.P. Mitalas. 1967. Calculation of Heat Transfer Functions for Multi-Layer Slabs. *ASHRAE Transactions* 73(1): III 1.1-1.7.
- Stephenson, D.G. and G.P. Mitalas. 1971. Calculation of Heat Transfer Functions for Multi-Layer Slabs. *ASHRAE Transactions* 77(2): 117-126.