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Optimum Duct Design for Variable Air Volume Systems, Part 1: Problem Domain Analysis of VAV Duct Systems

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ABSTRACT

Current duct design methods for variable air volume (VAV) systems are based on the use of peak constant airflow. However, VAV systems operate much of the time at an off-peak load condition, and the impact of varying airflow rates to the sizing of duct systems has not been considered. This and a companion paper introduce an optimum duct design procedure for VAV systems to investigate the importance of the varying airflows to the system design. Hourly airflow requirements, part-load fan characteristics, and duct static pressure control are incorporated into the problem formulation. Constraints, such as discrete duct sizes and velocity limitations, are incorporated into the duct design procedure. In part 1, the domain of a VAV optimization problem is analyzed to define the problem characteristics and to suggest an optimization procedure. In part 2, the VAV duct design procedure is fully developed and applied to several VAV duct systems with different parameter values. The results are analyzed to compare duct design methods, and the effects of several factors that influence optimal design are investigated.

INTRODUCTION

The design of duct systems is an important factor for effective, energy-efficient, and comfortable heating, ventilating, and air-conditioning (HVAC) systems. Commonly utilized duct design procedures have been developed for CAV systems and are based on peak load design conditions, for which the flow rates are assumed to be constant for the entire year. Yet, the most common system type for commercial office buildings is the variable air volume (VAV) system. VAV duct systems are commonly designed using maximum airflows to zones as if they are CAV systems. However, the VAV system spends much of the time at off-peak load conditions, providing less than peak flow for many hours of the year. Conventional duct design methods do not account for the actual zone load profile. Consequently, VAV duct systems may not be designed optimally using current design methods. For this reason, duct design methods should be reconsidered for VAV systems.

Three duct design methods are presented in the *1997 ASHRAE Handbook—Fundamentals* (ASHRAE 1997): equal friction, static regain, and the T-method. Of the three, the T-method, introduced by Tsal et al. (1988a, 1998b), is the only optimization-based method. The T-method finds optimal duct sizes and fan size in order to minimize system life-cycle cost. The system life-cycle cost includes the initial ductwork cost based on optimum duct sizes and the year-round electrical energy cost of the fan. The initial cost of the fan is not included. The calculation procedure of the T-method consists of three main steps: system condensing, fan selection, and system expansion. In the first step, the entire duct system is condensed into a single duct section for finding the ratios of optimal pressure losses using sectional hydraulic characteristics. An optimal system pressure loss is found in the second step. In the third step, the system pressure is distributed throughout the system sections. The T-method's calculations are based on a fixed amount of airflow throughout the year to determine duct sizes, overall system pressure drop, and fan energy cost. However, in VAV systems, the airflow varies continuously throughout a year's operation; therefore, the fan power changes with varying airflow. Fan power is also influenced if static pressure at the end of the longest duct line is controlled. Spitler et al. (1986) investigated fan energy consumption for VAV systems and found, for some buildings,

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that a large number of hours may be spent at a minimum flow fraction. Obviously, spending a large number of hours at the minimum fraction makes a significant impact on the fan electricity consumption. System life-cycle cost defined in the T-method does not account for these varying airflows of VAV systems, and, thus, the T-method may give non-optimal values for VAV system optimization.

In this study, the system life-cycle cost accounts for the impact of varying airflow rates on fan energy consumption. The system life-cycle cost is minimized to find the optimal duct sizes and to select a fan. For comparison purposes, several example VAV systems are optimized using the T-method by selecting maximum airflows as design air volumes, and then they are optimized again using an optimization procedure that accounts for varying airflow rates. The VAV optimization method differs from the T-method in that duct sizes are selected as explicit design variables that have discrete values; part-load fan characteristics are considered to find fan efficiencies for different airflows; and duct static pressure control is incorporated into the operating cost calculation. As a preliminary step to finding a VAV duct design procedure, this paper examines the problem domain analysis of VAV duct systems in terms of duct sizes. Tsal and Behls (1986) analyzed a two-dimensional hypothetical CAV duct system using a scalar field technique, which is the graphical representation of the objective function in terms of pressure losses of duct sections. The analysis will reveal which type of optimization is required, local or global optimization, and, consequently, suggest a VAV optimization technique. A companion paper (Kim et al. 2002) covers the economic analysis of VAV systems in order to compare a VAV optimization technique with the T-method by simulating designed systems under VAV operation.

PROBLEM DEFINITION

The goal of optimal VAV system duct design is to determine duct sizes and select a fan that minimizes system life-cycle cost. The system life-cycle cost as introduced by Tsal et al. (1988a, 1998b) is made up of initial and operating costs. Since VAV systems have varying airflows to meet the different loads in the zone, and the fan is controlled to maintain the desired static pressure at the end duct section of the longest duct line, the fan must be modeled on an hourly basis to determine the fan electricity consumption and operating cost.

The VAV duct design problem is defined as follows:

Minimize

$$E = E_p \cdot PWEF + E_s \quad \text{(life-cycle cost)} \qquad (1)$$

Subject to

$$D_p - D_c \geq 0 \quad \text{(telescopic constraint)}$$

$$L_i \leq D_i \leq U_i \quad \text{(standard sizes within upper and lower limits)}$$

where

E	=	system life-cycle cost, \$
E_p	=	first year energy cost, \$
E_s	=	initial cost, \$
$PWEF$	=	present worth escalation factor, dimensionless
D_p	=	upstream duct section diameter, m (in.)
D_c	=	downstream duct section diameter, m (in.)
D_i	=	$[D_1, D_2, \ldots, D_n]^T = [D^d]^T$
$D^d \in R^d$	=	feasible subset of discrete duct sizes, m (in.)
n	=	number of duct sections
L_i and U_i	=	lower and upper bounds of duct section i, m (in.), due to velocity or geometric constraints

Initial Cost

The initial cost includes the cost of the installed ducts and the fan. The duct cost is determined as a function of the cost per unit area of duct surface. The cost of HVAC equipment, such as fittings, heating coil, and cooling coil, are considered constant and are not included in the objective function, except the fan cost. The initial cost is

$$E_s = E_{duct} + E_{fan} \qquad (2)$$

where

E_{duct} = duct cost, \$, and

E_{fan} = fan cost, \$.

Duct Cost

The duct size of each duct section is a discrete design variable selected from the nominal sizes limited to the manufacturer's standard increments. For a round duct, the duct cost is

$$E_{duct} = S_d \pi D L \qquad (3)$$

where

S_d = unit ductwork cost, including material and labor, \$/m^2 (\$/ft^2),

D = duct diameter, m (in.),

L = duct length, m (in.).

For a rectangular duct, the duct cost is

$$E_{duct} = 2S_d\,(H + W)\,L \qquad (4)$$

where

H = duct height, m (in.), and

W = duct width, m (in.).

Fan Selection and Cost

Since duct optimization involves searching among different duct sizes, a suitable fan must be selected for each configuration and included in the initial cost calculation. When selecting a fan, the following factors govern the size and type of fan to be selected:

- Peak airflow rate
- Static pressure at a peak volumetric flow rate
- Efficiency: select a fan that will deliver the required volume at the expected static pressure with minimum horsepower

For any given candidate duct configuration, a fan is selected that can meet the system pressure requirement at the annual maximum airflow. For the hour with the annual maximum airflow, the pressure drop for every duct path is calculated and the highest-pressure drop is selected as the one determining fan selection. The fan is selected from a range of fans. Starting from the smallest fan size, the system design point is checked against the fan operating range. If the fan cannot operate at that design point, the next larger size fan is entered for selection. After a suitable fan is selected, the system is simulated through a year's operation to ensure that the fan can meet every hour's airflow requirements while maintaining the desired static pressure. The fan selection process resolves itself into the following three steps:

1. Prepare fan performance data.
2. Select the smallest fan that satisfies the system design point.
3. Check fan selection by simulating the system for an entire year.

Only centrifugal fans are considered in the fan selection, and the fan cost including motor and drive can be found from Means (1998).

Operating Cost

The operating cost consists mainly of the electrical energy cost required by the fan and is represented by the present value of the monthly costs. A multitude of electrical rate structures may be encountered in practice. In this study, electrical energy cost is assumed to be based on a unit energy cost and a demand charge based on the annual peak electricity consumption. However, any rate structure could be incorporated, as energy consumption is calculated on an hourly basis. The first year energy cost is

$$E_p = \frac{1}{K\eta_m}\left(\sum_Y \frac{Q_{fan}P_{fan}E_c}{\eta_f} + \frac{Q_{fan,\,peak}P_{fan,\,peak}E_d}{\eta_{f,\,peak}}\right) \tag{5}$$

where

Q_{fan} = fan airflow rate, m^3/s (cfm)

P_{fan} = fan total pressure, Pa (in.wg)

Y = system operating time, h/yr

E_c = unit energy cost, \$/kWh

E_d = energy demand cost, \$/kW

η_f = fan shaft efficiency, dimensionless

η_m = motor-drive efficiency, dimensionless

K = dimensional constant, 10^{-3} kW / [(m^3/s)·(N/s)] or 1.1741×10^{-4}kW / [cfm·in.wg]

The cost of hourly energy consumption is added across a year's operation according to varying fan airflow rates and fan total pressures. The shaft efficiency η_f is the function of fan speed and airflow rate. The hourly shaft efficiency is computed from a fan performance equation and is used for computing the hourly energy cost. The motor efficiency η_m is assumed to be a constant. The electrical energy demand cost E_d is based on the customer's peak kilowatt demand.

The present worth escalation factor is

$$PWEF = \frac{[(1+AER)/(1+AIR)]^a - 1}{1 - [(1+AIR)/(1+AER)]} \tag{6}$$

where

AER = annual escalation rate, dimensionless

AIR = annual interest rate, dimensionless

a = amortization period, years

Duct Modeling

The duct size is used to calculate the pressure loss in a duct section. The pressure loss of a duct section is calculated using the Darcy-Weisbach equation:

$$\Delta P = \left(\frac{fL}{D_h} + \sum C\right)\frac{V^2\rho}{2g_c} \tag{7}$$

where

f = friction factor, dimensionless

L = duct length, m (in.)

D_h = hydraulic diameter, m (in.)

ΣC = the summation of local loss coefficients within the duct section

V = mean air velocity, m/s

ρ = air density, kg/m^3 (lb_m/ft^3)

g_c = dimensional constant, 1.0(kg.m)/($N.s^2$) [32.2(lb_mft)/(lb_f^2)]

For a rectangular duct, the equivalent-by-friction diameter (hydraulic diameter) is

$$D_f = 2\frac{H \cdot W}{H + W}. \tag{8}$$

Next, the equivalent length L_e is introduced,

$$L_e = L + \frac{D_h}{f}\sum C. \tag{9}$$

and substituted into the Darcy-Weisbach equation to yield

$$\Delta P = \frac{f}{D_h}L_e\frac{\rho V^2}{2g_c}. \tag{10}$$

The friction factor in Equation 10 is calculated from Altshul's equation:

$$f = 0.11\left(\frac{\varepsilon}{D_h} + \frac{68}{R_e}\right) \quad (11)$$

And the Reynolds number is given by

$$\text{Re} = \frac{D_h V}{\nu} \quad (12)$$

where

ν = kinematic viscosity, m^2/s (ft^2/s).

Fan Modeling

The fan model was introduced for estimating airflows as a component of fluid flow networks (Clark 1985; Walton 1989). It uses fourth-order polynomial fits to the dimensionless head and efficiency to predict the pressure rise and power consumption. The fan similarity laws allow the dimensionless curves to be used to treat varying rotation speed and different diameters. The performance of a fan is characterized in terms of the pressure rise across the device and the shaft power requirements at a given fluid flow rate. These two characteristics are pressure head and efficiency. The two dimensionless performance curves that relate pressure head and efficiency to fluid flow rate are represented by polynomials with empirical coefficients that can be computed using manufacturer's data. These performance curves form the basis of the model.

In order to simplify and generalize the model, the dimensionless variables are defined using the fan similarity laws: flow coefficient and pressure head coefficient.

The dimensionless flow coefficient is defined as

$$\phi = \frac{\dot{m}}{\rho N d^3} \quad (13)$$

where

$\dot{m}$ = dry air mass flow rate, kg/s

ρ = entering moist air density, kg/m^3

N = fan speed, rps

d = fan wheel diameter, m

The dimensionless pressure head coefficient is defined as

$$\psi = \frac{\Delta P}{\rho N^2 d^2} \quad (14)$$

where

ΔP = pressure rise across the fan, Pa.

It should be noticed that the use of dimensionless variables implicitly applies the fan laws for changes in speed, density, and diameter.

The fan efficiency is defined as

$$\eta_s = \frac{\dot{m} \cdot \Delta P}{\rho \cdot \dot{W}_s} \quad (15)$$

where

$\dot{W}_s$ = shaft power, W.

The polynomial performance curves are

$$\psi = a_0 + a_1\phi + a_2\phi^2 + a_3\phi^3 + a_4\phi^4 \quad (16)$$

$$\eta_f = b_0 + b_1\phi + b_2\phi^2 + b_3\phi^3 + b_4\phi^4 \quad (17)$$

where a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, and b_4 are determined from the manufacturer's data.

The total system power that is used to calculate the operating cost is expressed in terms of the shaft power and the motor efficiency.

$$\dot{W}_t = \frac{\dot{W}_s}{\eta_m} \quad (18)$$

Operating Cost Calculation

Variable air volume systems require efficient, stable operation over the entire airflow range. Since VAV systems seldom operate at full design air volume, part-load power savings at reduced air volume are realized significantly through the proper fan operation. Fans are controlled by placing a static pressure sensor in the downstream ductwork, typically two-thirds of the way down the longest duct path. This sensor is set at a static pressure that will ensure sufficient pressure to move the air from that point through the remaining ductwork. For instance, in response to a decreasing cooling load, static pressure in the ductwork increases. The static pressure sensor detects an increase in duct pressure and signals the fan to decrease speed until the static pressure is satisfied. According to Englander and Norford (1992a, 1992b), the set point of static pressure is recommended at 1.5 in. wg for September through May and 2.5 in. wg for June through August. Since the fan operation is changed with the varying airflow, the fan model is used to get the information of changed operating points. In addition to the change of fan operation, fan efficiency is also an important factor for the calculation of the fan operating cost. The required shaft power input is greater than the power input to the air because of irreversibilities. The ratio of the air power to the shaft power is the fan efficiency. A fan has different efficiency with a change in airflow requirements. For a given air volume and pressure requirement, a corresponding fan speed must be found to calculate the efficiency at that operating point and, consequently, the fan operating cost. Hourly fan operating cost is summed for a year and then added to the initial cost to find the system life-cycle cost.

Using the fan model, fan power can be calculated for a given air flow and pressure rise. The dimensionless performance curve relating pressure head and air volume includes fan speed (Equation 16). The equation is rearranged with respect to fan speed, and the fan speed at a given air flow and pressure can be found using a numerical root-finding method. Once the fan speed is found, the shaft efficiency is obtained from Equation 17. The shaft power is computed with the resulting efficiency. Consequently, the total power consumption is computed with motor efficiency. The algorithm to calculate the fan power is as follows:

1. Compute the system pressure loss for a given airflow rate (Q, ΔP)
2. Find the fan speed (rpm)
3. Rearrange Equation 16 with respect to fan speed:

$$F(N) = a_0 + a_1\phi(N) + a_2\phi^2(N) + a_3\phi^3(N) + a_4\phi^4(N) - \psi(N) \qquad (19)$$

4. Set $F(N) = 0$ and find the fan speed using numerical methods
5. Compute the fan efficiency η_s using Equation 17
6. Compute the shaft power:

$$\dot{W}_s = \frac{\Delta P \cdot Q}{\eta_s} \qquad (20)$$

7. Compute the total system power:

$$\dot{W}_m = \frac{\dot{W}_s}{\eta_m} \qquad (21)$$

APPROACH FOR THE PROBLEM DOMAIN ANALYSIS

The questions that arise in optimization problems are convexity, differentiability of convex functions, and local or global optimum points. Arora (1989) stated, "If $f(x)$ is a local minimum for a convex function $f(x^*)$ on a convex set S, then it is also a global minimum. The convexity of the objective function is defined if and only if the Hessian matrix of the function is positive definite at all points in the convex set S." However, in duct design problems, the Hessian matrix of the objective function cannot be derived since the function is not differentiable with respect to the duct size. The objective function is composed of several procedures that must be evaluated consecutively. Because the use of a convexity test is impossible, one has to use other approaches to analyze the duct design problem, such as exhaustive search and graphical representation. However, it may be impossible to definitively prove that there is a global minimum if the design space cannot be tested for convexity.

For the purpose of characterization and computation of local or global minima of the duct design problem, the analysis of the problem domain is as follows:

- Exhaustive search of the problem domain: An exhaustive search computes the function value at all discrete points in the problem domain and obtains any plausible local minima by comparing every point with its neighborhood. The neighborhood of N of the point $\mathbf{x}^*$ is mathematically defined as the set of points

$$N = \{\mathbf{x} \mid \mathbf{x} \in S \text{ with } \|\mathbf{x} - \mathbf{x}^*\| < \delta\} \text{ for some small } \delta > 0.$$

- In the exhaustive search, δ is set to a search discrete step size of Δd. Thus, the neighborhood is any point where any coordinate of a point has $d \pm \Delta d$. In one-dimensional space, the neighborhood of a point comprises two points. In two-dimensional space, there are eight neighbor points. Generally, n-dimension has $3^n - 1$ neighbors.
- If the exhaustive search finds several local minima with a small Δd, then those local minima should be tested as to whether they are truly local minima by searching each neighborhood with a smaller discrete step size. If the search shows that it has one global minimum, the minimum point can be accepted as an optimal solution with the interval of uncertainty of a discrete step size, and the direct search method can be applied to the duct design problem.
- Employment of a direct optimization method, such as the Nelder and Mead downhill simplex method: Assuming one global minimum exists in the design domain, a direct optimization method can be applied to the duct design problem. A starting point is chosen at an extreme point, such as the corner point of the boundary feasible region, and the search result is compared to the one obtained with different starting points. The downhill simplex method (Nelder and Mead 1965) is simple in calculation and uncomplicated in logic. The method is also effective when evaluation errors are significant because it operates on the worst rather than the best point (Reklaitis et al. 1983). Thus, in this study, the downhill simplex method is applied to obtain an optimal design point.
- With a starting point $\mathbf{P}_0$, the initial simplex takes the other points using

$$\mathbf{P}_i = \mathbf{P}_0 + \lambda\mathbf{e}_i.$$

- The steps that are taken in the simplex method are reflections, expansions, and contractions. The simplex is reflected away from the high point, and, if possible, it is expanded away on one or another direction to take larger steps. When it reaches a valley floor, the simplex is contracted in the transverse direction and oozes down the valley. The termination criterion used for this study is the rate of the difference to the sum of the minimum and maximum function values of the simplex. One thing to be noted is that the criteria might be fooled by a single anomalous step that failed to make progress, so it is recommended that the minimization routine be restarted at a point where the current step is at a minimum (Press et al. 1992).

COMPUTATION RESULTS AND DISCUSSION

An Example Duct System for Analysis

A hypothetical duct system shown in Figure 1 was selected to study the problem domain analysis. The system originated from the five-section duct system of Tsal et al. (1988a, 1998b), in which duct sections 3 and 4 are eliminated to create a three-section duct system. For the newly

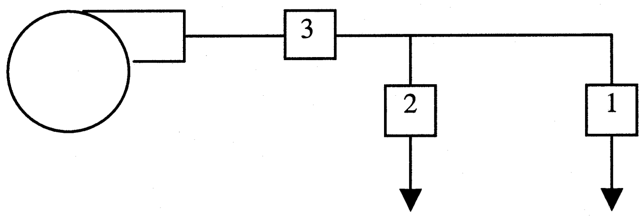


Figure 1 *Hypothetical three-section duct system.*

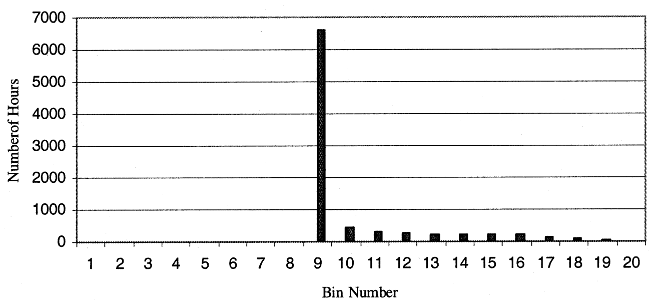


Figure 2 *Annual distribution of fraction of full flow for the hypothetical system.*

created duct system, the duct diameter for section 2 is presized to 0.178 m (7 in.) and sections 1 and 3 are round and unsized. Now the system becomes a two-dimensional three-section duct system. The system parameters are summarized in Tables 1, 2, and 3, and they are economic, general, and sectional data, respectively. In Table 3, the peak airflows at terminal duct sections 1 and 2 are 0.70 m^3/s and 0.50 m^3/s, respectively. It is defined to have variable airflows and to operate a full year of 8760 hours. The minimum fraction of full flow is set to 0.4. Figure 2 shows the fractional flow distributions for this hypothetical duct system. The fractions of full flow are divided into increments of 0.05. Bin 1 corresponds to zero flow, bin 9 to minimum fraction, and bin 20 to the full flow.

Exhaustive Search

When the exhaustive search was applied to the example system with a discrete step size of one inch, a design point was identified as that having neighboring points with higher functional values. This point may be called an apparent local minimum. "Apparent" because, while it appears to be a local minimum, it has not yet been established whether or not it is a local minimum. The design values were 10 in. and 11 in. in ducts 1 and 3, respectively, and the system life-cycle cost was $4191.36. The problem domain was searched again with a discrete step size of 0.1 in., and a design point was identified as a local minimum. An apparent local minimum appeared at 9.6 in. and 11.4 in. in ducts 1 and 3, respectively, and the system life-cycle cost was $4185.90. When the step size was further subdivided into 0.01 in., the domain had an apparent local minimum at 9.63 in. and 11.43 in. in ducts 1 and 3, respectively, and the system life-cycle cost was $4185.88. A very likely global minimum was found at an apparent local minimum with the exhaustive search.

Graphical Representation

For a visual inspection of the problem domain, contour maps were drawn on two- and three-dimensional spaces for 0.1-in. and 0.01-in. grid searches, as shown in Figures 3 and 4. It can be seen that the objective function has a typical shape of the convex function. The surface does not have a deep

TABLE 1
Economic Data

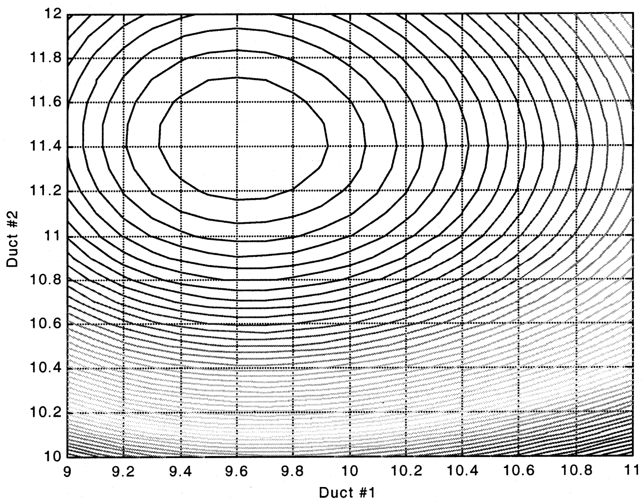
Energy cost (E_p)	2.03 ¢/kWh
Energy demand cost	13 $/kW
Duct cost (S_d)	43.27 $/m^2 (4.02 $/ft^2)
System operation time (Y)	8760 h/yr
Present worth escalation factor (PWEF)	8.61

TABLE 2
General Data

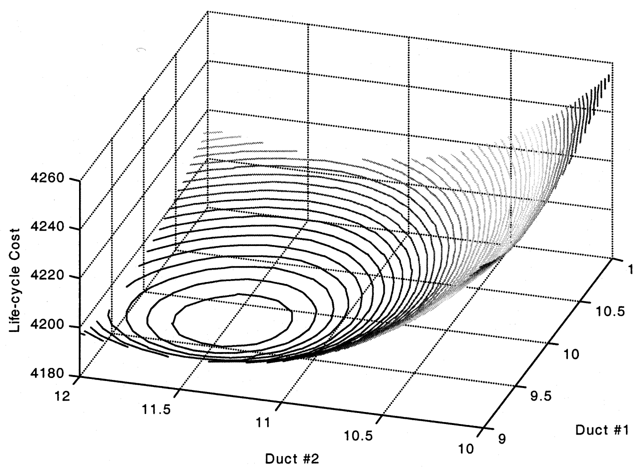
Data	SI units	I-P units
Air temperature (t)	22°C	71.6°F
Absolute roughness of aluminum duct (ε)	0.0003 m	0.00098 ft
Kinematic viscosity (ν)	1.54×10^{-5} m^2/s	1.66×10^{-4} ft^2/s
Air density (ρ)	1.2 kg/m^3	0.75 lb/ft^3
Motor efficiency (η_e)	0.75	0.75
Total system airflow (Q_{fan})	1.42 m^3/s	3010 cfm

TABLE 3
Sectional Data of Three-Section Duct System

Sections			Peak Air Flow	Duct Length	Additional Pressure Loss	
Sec.	Ch1	Ch2	cfm (L/s)	ft (m)	in. water (Pa)	C-coefficient
1	0	0	1483 (700)	45.9 (14.0)	0.10 (25.0)	0.80
2	0	0	466 (220)	39.3 (12.0)	0.15 (37.5)	0.65
3	1	2	1950 (920)	65.0 (19.8)	0.49 (121.8)	1.50

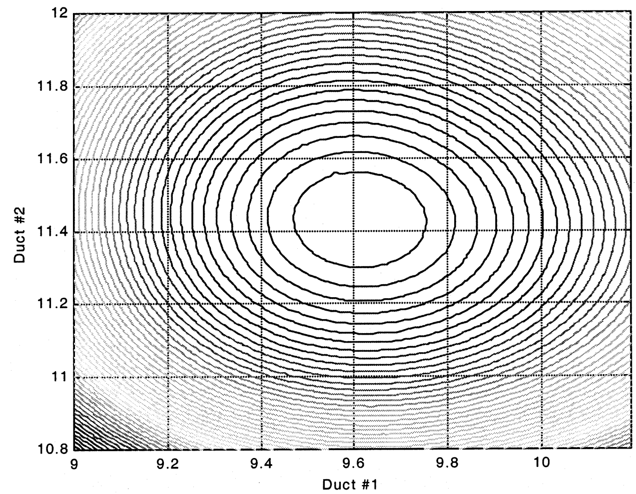


(a) Two-dimensional contour map

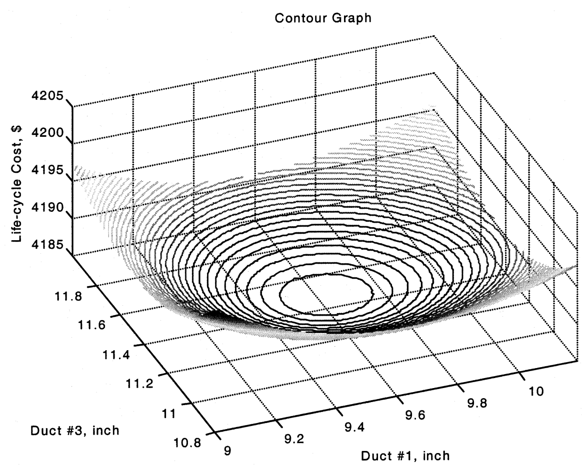


(b) Three-dimensional contour map

Figure 3 *Contour map of system life-cycle cost of the hypothetical duct system with 0.1-in. exhaustive search.*



(a) Two-dimensional contour map



(b) Three-dimensional contour map

Figure 4 *Contour map of system life-cycle cost of the hypothetical duct system with 0.01-in. exhaustive search.*

valley and a steep slope. The function slopes gradually down to the bottom. From Figures 3 and 4, one can easily find that the optimum solution lies around 9.6 in. and 11.4 in. in ducts 1 and 3, respectively. The graphical representation also suggests that the problem very likely has only a global minimum.

The Nelder and Mead Downhill Simplex Method

In a two-dimensional exhaustive search, it was found that the problem domain had a very likely global minimum. Next, the downhill simplex method was applied to the hypothetical system. The initial starting point for a search was chosen at an extreme point, such as the corner point of the bounded region. The other N points of a simplex are defined by

$$\mathbf{P}_\mathrm{i} = \mathbf{P}_0 + 1.0\ \mathbf{e}_i,$$

where the $\mathbf{e}_i$ is an N unit vector, and 1.0 is the problem's characteristic length scale. The fractional convergence tolerance of the function value for a simplex routine was set to 1.0e-8. The function tolerance to stop routine iteration was set to 0.001. The result is shown at Table 4.

Depending on the function tolerance, one can obtain more significant digits in the optimum value. However, the number of function evaluations was increased. Considering that the

TABLE 4
Global Minimum Found with the Downhill Simplex Method

Starting Point (duct 1, duct 3)	Duct 1, in. (mm)	Duct 3, in.	Life-Cycle Cost, $	Number of Function Evaluation
Lower left (9, 10)	9.61 (244.1)	11.43 (290.3)	4185.883	88
Upper right (21, 24)	9.62 (244.3)	11.44 (290.6)	4185.881	174
Upper left (9, 24)	9.62 (244.3)	11.44 (290.6)	4185.881	184
Lower right (21, 10)	9.61 (244.1)	11.43 (290.3)	4185.883	113

original optimum is discrete, it is probably unnecessary to increase the fractional convergence tolerance. With the function tolerance of 1.0e-8, two significant digits were acquired in duct sizes and three significant digits in function values. With the function tolerance of 1.0e-6, one significant digit was acquired in duct sizes and two significant digits in function values. The problem's characteristic length scale also affects the number of function evaluations and the iterations. The appropriate values range from 1.0 to 1.5. When the length scale is smaller, the number of function evaluations was increased to more than 1000. Repeating the routine at a point that is found to be an optimum until the function value is not further improved was also necessary. The reason for routine iteration is to avoid being trapped at a point that is not a minimum due to anomalous steps in moving the simplex point. Occasionally, after a certain number of steps, the simplex took a series of continuous contractions, and the movement of the simplex was stopped at a certain point. When the program routine was repeated at a trapped point, the simplex easily got out of the trapped point and settled down at a global minimum. In order to be certain that the trapped point was not a local minimum, its neighborhood was exhaustively searched with a 0.0001-in. step size. No point was found surrounded by higher function values. This proves that the trapped point is not a local minimum.

CONCLUSIONS

Exhaustive search, graphical representation, and the Nelder and Mead's downhill simplex method have been applied in this paper to a hypothetical two-dimensional VAV duct system to define the characteristics of the problem domain. Consideration has been given to the verification of the local or global minimum and to the shape of the objective function when duct sizes are design variables. The exhaustive search has been done with 1-in., 0.1-in., and 0.01-in. increments. It has found only one local minimum in the two-dimensional problem domain. Contour maps of the life-cycle cost show that the domain does not have a deep valley. The function slopes gradually down to the bottom. The Nelder and Mead's downhill simplex algorithm also found a similar location of the global minimum as the exhaustive search in the two-dimensional duct systems. The test results also suggest the function tolerance of 1.0e-6 and the problem's characteristic length λ from 1.0 to 1.5. Considering the results from the above three approaches, it can be concluded that the problem very likely has only a global minimum, and the optimal point can be found using a direct search method.

In the companion paper (Kim et al. 2002), the Nelder and Mead downhill simplex method with a discrete programming algorithm is applied to several VAV duct systems to find optimum duct sizes and a fan. The results are compared to those derived from the T-method for economic analysis.

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